

# Coordinate Geometry

## Exercise 7.1

### Question 1:

Find the distance between the following pairs of points:

(i) (2, 3), (4, 1) (ii) (-5, 7), (-1, 3) (iii) (a, b), (-a, -b)

### Answer:

(i) Distance between the two points is given by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Therefore, distance between (2, 3) and (4, 1) is given by

$$\begin{aligned} l &= \sqrt{(2-4)^2 + (3-1)^2} = \sqrt{(-2)^2 + (2)^2} \\ &= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \end{aligned}$$

(ii) Distance between (-5, 7) and (-1, 3) is given by

$$\begin{aligned} l &= \sqrt{(-5-(-1))^2 + (7-3)^2} = \sqrt{(-4)^2 + (4)^2} \\ &= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \end{aligned}$$

(iii) Distance between (a, b) and (-a, -b) is given by

$$\begin{aligned} l &= \sqrt{(a-(-a))^2 + (b-(-b))^2} \\ &= \sqrt{(2a)^2 + (2b)^2} = \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2} \end{aligned}$$

### Question 2:

Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.

**Answer:**

Distance between points  $(0,0)$  and  $(36,15)$

$$\begin{aligned} &= \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{36^2 + 15^2} \\ &= \sqrt{1296 + 225} = \sqrt{1521} = 39 \end{aligned}$$

Yes, we can find the distance between the given towns A and B.

Assume town A at origin point  $(0, 0)$ .

Therefore, town B will be at point  $(36, 15)$  with respect to town A.

And hence, as calculated above, the distance between town A and B will be

39 km.

**Question 3:**

**Determine if the points  $(1, 5)$ ,  $(2, 3)$  and  $(-2, -11)$  are collinear.**

**Answer:**

Let the points  $(1, 5)$ ,  $(2, 3)$ , and  $(-2, -11)$  be representing the vertices A, B, and C of the given triangle respectively.

Let  $A = (1, 5), B = (2, 3), C = (-2, -11)$

$$\therefore AB = \sqrt{(1-2)^2 + (5-3)^2} = \sqrt{5}$$

$$BC = \sqrt{(2-(-2))^2 + (3-(-11))^2} = \sqrt{4^2 + 14^2} = \sqrt{16 + 196} = \sqrt{212}$$

$$CA = \sqrt{(1-(-2))^2 + (5-(-11))^2} = \sqrt{3^2 + 16^2} = \sqrt{9 + 256} = \sqrt{265}$$

Since  $AB + BC \neq CA$ ,

Therefore, the points (1, 5), (2, 3), and (-2, -11) are not collinear.

**Question 4:**

**Check whether (5, -2), (6, 4) and (7, -2) are the vertices of an isosceles triangle.**

**Answer:**

Let the points (5, -2), (6, 4), and (7, -2) are representing the vertices A, B, and C of the given triangle respectively.

$$AB = \sqrt{(5-6)^2 + (-2-4)^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$$

$$BC = \sqrt{(6-7)^2 + (4-(-2))^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37}$$

$$CA = \sqrt{(5-7)^2 + (-2-(-2))^2} = \sqrt{(-2)^2 + 0^2} = 2$$

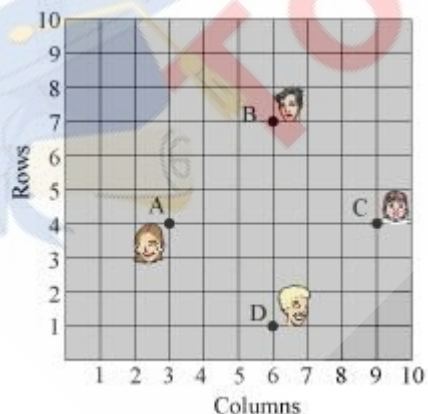
Therefore,  $AB = BC$

As two sides are equal in length, therefore, ABC is an isosceles triangle.

**Question 5:**

**In a classroom, 4 friends are seated at the points A, B, C and D as shown in the following figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees.**

**Using distance formula, find which of them is correct.**



**Answer:**

It can be observed that A (3, 4), B (6, 7), C (9, 4), and D (6, 1) are the positions of these 4 friends.

$$AB = \sqrt{(3-6)^2 + (4-7)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

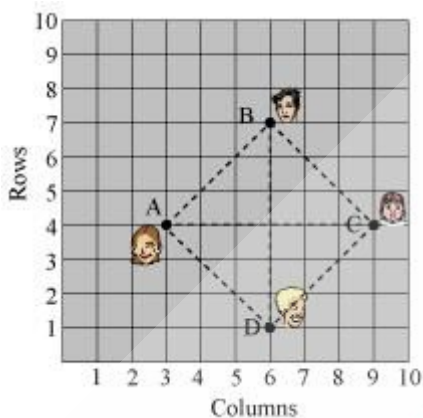
$$BC = \sqrt{(6-9)^2 + (7-4)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CD = 9-6+4-12 = 32+32 = 9+9 = 18 = 32$$

$$AD = \sqrt{(3-6)^2 + (4-1)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$\text{Diagonal AC} = \sqrt{(3-9)^2 + (4-4)^2} = \sqrt{(-6)^2 + 0^2} = 6$$

$$\text{Diagonal BD} = \sqrt{(6-6)^2 + (7-1)^2} = \sqrt{0^2 + (6)^2} = 6$$



It can be observed that all sides of this quadrilateral ABCD are of the same length and also the diagonals are of the same length.

Therefore, ABCD is a square and hence, Champa was correct

#### Question 6:

Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) (-1, -2), (1, 0), (-1, 2), (-3, 0)

(ii) (-3, 5), (3, 1), (0, 3), (-1, -4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

**Answer:**

(i) Let the points  $(-1, -2)$ ,  $(1, 0)$ ,  $(-1, 2)$ , and  $(-3, 0)$  be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$\therefore AB = \sqrt{(-1-1)^2 + (-2-0)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{(1-(-1))^2 + (0-2)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$CD = \sqrt{(-1-(-3))^2 + (2-0)^2} = \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$AD = \sqrt{(-1-(-3))^2 + (-2-0)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\text{Diagonal AC} = \sqrt{(-1-(-1))^2 + (-2-2)^2} = \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$$

$$\text{Diagonal BD} = \sqrt{(1-(-3))^2 + (0-0)^2} = \sqrt{(4)^2 + 0^2} = \sqrt{16} = 4$$

It can be observed that all sides of this quadrilateral are of the same length and also, the diagonals are of the same length. Therefore, the given points are the vertices of a square.

(ii) Let the points  $(-3, 5)$ ,  $(3, 1)$ ,  $(0, 3)$ , and  $(-1, -4)$  be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$AB = \sqrt{(-3-3)^2 + (5-1)^2} = \sqrt{(-6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$

$$BC = \sqrt{(3-0)^2 + (1-3)^2} = \sqrt{(3)^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$$

$$CD = \sqrt{(0-(-1))^2 + (3-(-4))^2} = \sqrt{(1)^2 + (7)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$AD = \sqrt{(-3-(-1))^2 + (5-(-4))^2} = \sqrt{(-2)^2 + (9)^2} = \sqrt{4+81} = \sqrt{85}$$

It can be observed that all sides of this quadrilateral are of different lengths. Therefore, it can be said that it is only a general quadrilateral, and not specific such as square, rectangle, etc.

(iii) Let the points (4, 5), (7, 6), (4, 3), and (1, 2) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$AB = \sqrt{(4-7)^2 + (5-6)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$BC = \sqrt{(7-4)^2 + (6-3)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$CD = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$AD = \sqrt{(4-1)^2 + (5-2)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$\text{Diagonal AC} = \sqrt{(4-4)^2 + (5-3)^2} = \sqrt{(0)^2 + (2)^2} = \sqrt{0+4} = 2$$

$$\text{Diagonal BD} = \sqrt{(7-1)^2 + (6-2)^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$

It can be observed that opposite sides of this quadrilateral are of the same length. However, the diagonals are of different lengths. Therefore, the given points are the vertices of a parallelogram.

### Question 7:

Find the point on the x-axis which is equidistant from (2, -5) and (-2, 9).

**Answer:**

We have to find a point on x-axis. Therefore, its y-coordinate will be 0.

Let the point on x-axis be  $(x, 0)$ .

$$\text{Distance between } (x, 0) \text{ and } (2, -5) = \sqrt{(x-2)^2 + (0-(-5))^2} = \sqrt{(x-2)^2 + (5)^2}$$

$$\text{Distance between } (x, 0) \text{ and } (-2, 9) = \sqrt{(x-(-2))^2 + (0-(-9))^2} = \sqrt{(x+2)^2 + (9)^2}$$

By the given condition, these distances are equal in measure.

$$\begin{aligned}\sqrt{(x-2)^2 + (5)^2} &= \sqrt{(x+2)^2 + (9)^2} \\ (x-2)^2 + 25 &= (x+2)^2 + 81 \\ x^2 + 4 - 4x + 25 &= x^2 + 4 + 4x + 81 \\ 8x &= 25 - 81 \\ 8x &= -56 \\ x &= -7\end{aligned}$$

Therefore, the point is  $(-7, 0)$ .

### Question 8:

Find the values of  $y$  for which the distance between the points  $P(2, -3)$  and  $Q(10, y)$  is 10 units.

**Answer:**

It is given that the distance between  $(2, -3)$  and  $(10, y)$  is 10.

$$\begin{aligned}\text{Therefore, } \sqrt{(2-10)^2 + (-3-y)^2} &= 10 \\ \sqrt{(-8)^2 + (3+y)^2} &= 10 \\ 64 + (y+3)^2 &= 100 \\ (y+3)^2 &= 36 \\ y+3 &= \pm 6 \\ y+3 &= 6 \text{ or } y+3 = -6 \\ \text{Therefore, } y &= 3 \text{ or } -9\end{aligned}$$

### Question 9:

If  $Q(0, 1)$  is equidistant from  $P(5, -3)$  and  $R(x, 6)$ , find the values of  $x$ . Also find the distance  $QR$  and  $PR$ .

**Answer:**

$$PQ = QR$$

$$\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(0-x)^2 + (1-6)^2}$$

$$\sqrt{(5)^2 + (-4)^2} = \sqrt{(-x)^2 + (-5)^2}$$

$$\sqrt{25+16} = \sqrt{x^2 + 25}$$

$$41 = x^2 + 25$$

$$16 = x^2$$

$$x = \pm 4$$

Therefore, point R is (4, 6) or (-4, 6).

When point R is (4, 6),

$$PR = \sqrt{(5-4)^2 + (-3-6)^2} = \sqrt{1^2 + (-9)^2} = \sqrt{1+81} = \sqrt{82}$$

$$QR = \sqrt{(0-4)^2 + (1-6)^2} = \sqrt{(-4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

When point R is (-4, 6),

$$PR = \sqrt{(5-(-4))^2 + (-3-6)^2} = \sqrt{(9)^2 + (-9)^2} = \sqrt{81+81} = 9\sqrt{2}$$

$$QR = \sqrt{(0-(-4))^2 + (1-6)^2} = \sqrt{(4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

#### Question 10:

Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (-3, 4).

**Answer:**

Point (x, y) is equidistant from (3, 6) and (-3, 4).

$$\therefore \sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x-(-3))^2 + (y-4)^2}$$

$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$$

$$(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$$x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$$

$$36 - 16 = 6x + 6x + 12y - 8y$$

$$20 = 12x + 4y$$

$$3x + y = 5$$

$$3x + y - 5 = 0$$

