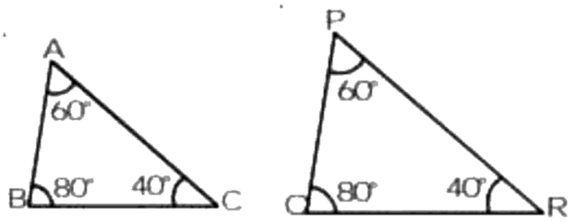
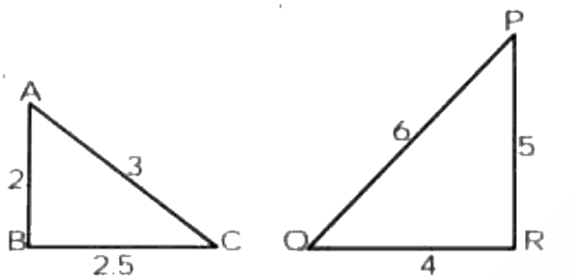
Ex - 6.3

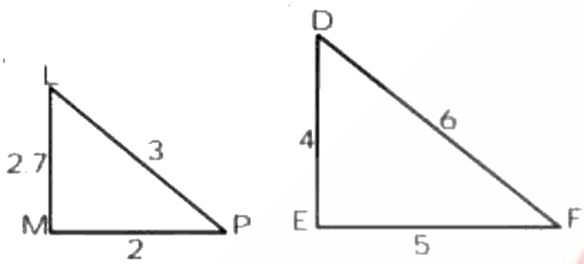
Q1. State which pairs of triangles in figure, are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :



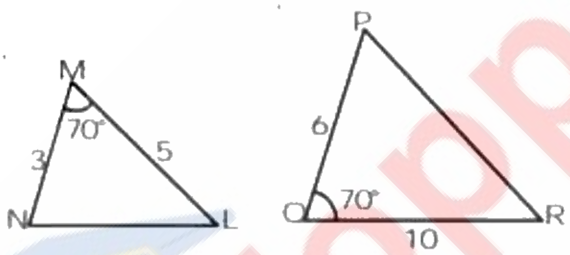
(i)



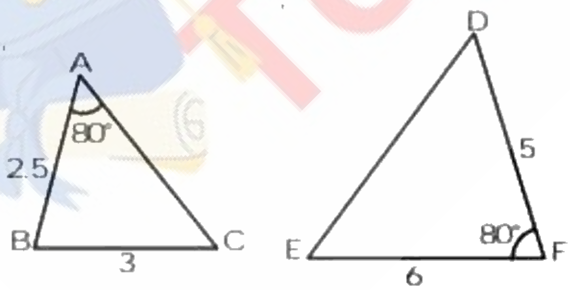
(ii)



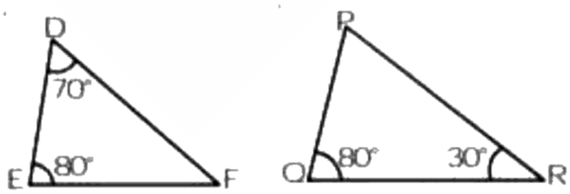
(iii)



(iv)



(v)



(vi)

Sol. (i) Yes. $\angle A = \angle P = 60°$, $\angle B = \angle Q = 80°$,

$\angle C = \angle R = 40°$

Therefore, $\Delta ABC \sim \Delta PQR$.

By AAA similarity criterion

(ii) Yes.

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}, \frac{BC}{RP} = \frac{2.5}{5} = \frac{1}{2}, \frac{CA}{PQ} = \frac{3}{6} = \frac{1}{2}$$

Therefore, $\Delta ABC \sim \Delta QRP$.

By SSS similarity criterion.

(iii) No.

$$\frac{MP}{DE} = \frac{2}{4} = \frac{1}{2}, \frac{LP}{DF} = \frac{3}{6} = \frac{1}{2}, \frac{LM}{EF} = \frac{2.7}{5} \neq \frac{1}{2}$$

i.e., $\frac{MP}{DE} = \frac{LP}{DF} \neq \frac{LM}{EF}$

Thus, the two triangles are not similar.

(iv) Yes,

$$\frac{MN}{OP} = \frac{ML}{QR} = \frac{1}{2}$$

and $\angle NML = \angle PQR = 70°$

By SAS similarity criterion

$\Delta NML \sim \Delta PQR$

(v) No,

$$\frac{AB}{FD} \neq \frac{AC}{FE}$$

Thus, the two triangles are not similar

(vi) In triangle DEF $\angle D + \angle E + \angle F = 180°$

$70° + 80° + \angle F = 180°$

$\angle F = 30°$

In triangle PQR

$\angle P + 80° + 30° = 180°$

$\angle P = 70°$

$\angle E = \angle Q = 80°$

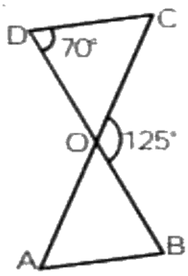
$\angle D = \angle P = 70°$

$\angle F = \angle R = 30°$

By AAA similarity criterion,

$\Delta DEF \sim \Delta PQR$.

Q2. In figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$ and $\angle CDO = 70°$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.



Sol. From figure,

$\angle DOC + 125° = 180°$

$\Rightarrow \angle DOC = 180° - 125° = 55°$

$\angle DCO + \angle CDO + \angle DOC = 180°$

(Sum of three angles of ΔODC)

$\Rightarrow \angle DCO + 70° + 55° = 180°$

$\Rightarrow \angle DCO + 125° = 180°$

$\Rightarrow \angle DCO = 180° - 125° = 55°$

Now, we are given that $\Delta ODC \sim \Delta OBA$

$\Rightarrow \angle OCD = \angle OAB$

$\Rightarrow \angle OAB = \angle OCD = \angle DCO = 55°$

i.e., $\angle OAB = 55°$

Hence, we have

$\angle DOC = 55°$, $\angle DCO = 55°$, $\angle OAB = 55°$

Q3. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

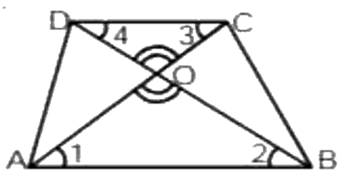
Sol. In figure, AB || DC

$\Rightarrow \angle 1 = \angle 3, \angle 2 = \angle 4$

(Alternate interior angles)

Also $\angle DOC = \angle BOA$

(Vertically opposite angles)

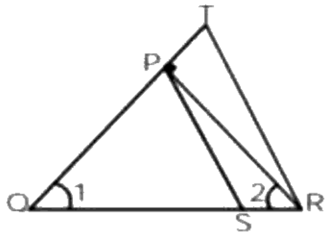


$\Rightarrow \Delta OCD \sim \Delta OAB \quad \Rightarrow \frac{OC}{OA} = \frac{OD}{OB}$

(Ratios of the corresponding sides of the similar triangle)

$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$ (Taking reciprocals)

Q4. In figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.



Sol. In figure, $\angle 1 = \angle 2$ (Given)

$\Rightarrow$ PQ = PR

(Sides opposite to equal angles of ΔPQR)

We are given that

$$\frac{QR}{QS} = \frac{QT}{PR}$$

$$\Rightarrow \frac{QR}{QS} = \frac{QT}{PQ} \quad (\because PQ = PR \text{ proved})$$

$$\Rightarrow \frac{QS}{QR} = \frac{PQ}{QT} \quad \text{(Taking reciprocals)} \; ...(1)$$

Now, in ΔPQS and ΔTQR, we have

$\angle PQS = \angle TQR$ (Each $= \angle 1$)

and $\frac{QS}{QR} = \frac{PQ}{QT}$ (By (1))

Therefore, by SAS similarity criterion, we have

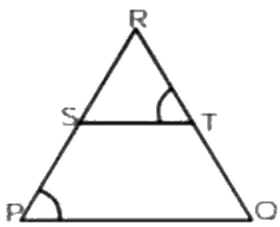
$\Delta PQS \sim \Delta TQR$.

Q5. S and T are points on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

Sol. In figure, We have ΔRPQ and ΔRTS in which

$\angle RPQ = \angle RTS$ (Given)

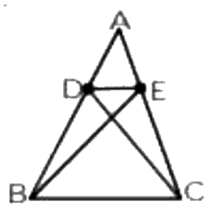
$\angle PRQ = \angle SRT$ (Each $= \angle R$)



Then by AA similarity criterion, we have

$\Delta RPQ \sim \Delta RTS$

Q6. In figure, if $\Delta ABE \cong \Delta ACD$, show that $\Delta ADE \sim \Delta ABC$.



Sol. In figure,

$\Delta ABE \cong \Delta ACD$ (Given)

$\Rightarrow AB = AC$ and $AE = AD$ (CPCT)

$\Rightarrow \frac{AB}{AC} = 1$ and $\frac{AD}{AE} = 1$

$\Rightarrow \frac{AB}{AC} = \frac{AD}{AE}$ (Each = 1)

Now, in ΔADE and ΔABC, we have

$\frac{AD}{AE} = \frac{AB}{AC}$ (proved)

i.e., $\frac{AD}{AB} = \frac{AE}{AC}$

and also $\angle DAE = \angle BAC$ (Each = $\angle A$)

$\Rightarrow \Delta ADE \sim \Delta ABC$(By SAS similarity criterion)

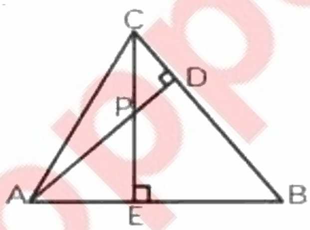
Q7. In figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that :

(i) $\Delta AEP \sim \Delta CDP$

(ii) $\Delta ABD \sim \Delta CBE$

(iii) $\Delta AEP \sim \Delta ADB$

(iv) $\Delta PDC \sim \Delta BEC$



Sol. (i) In ΔAEP and ΔCDP,

$\angle APE = \angle CPD$ (vertically opposite angles)

$\angle AEP = \angle CDP = 90°$

$\therefore$ By AA similarity

$\Delta AEP \sim \Delta CDP$

(ii) In ΔABD and ΔCBE,

$\angle ABD = \angle CBE$ (common)

$\angle ADB = \angle CEB = 90°$

$\therefore$ By AA similarity

$\Delta ABD \sim \Delta CBE$

(iii) In ΔAEP and ΔADB,

$\angle PAE = \angle DAB$ (common)

$\angle AEP = \angle ADB = 90°$

∴ By AA similarity

$\Delta AEP \sim \Delta ADB$

(iv) In ΔPDC and ΔBEC,

$\angle PCD = \angle BCE$ (common)

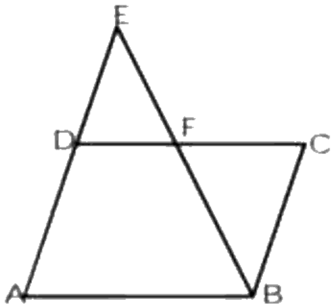
$\angle PDC = \angle BEC = 90^\circ$

∴ By AA similarity

$\Delta PDC \sim \Delta BEC$

Q8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$.

Sol.



In ΔABE and ΔCFB,

$\angle EAB = \angle BCF$ (opp. angles of parallelogram)

$\angle AEB = \angle CBF$ (Alternate interior angles. As $AE \parallel BC$)

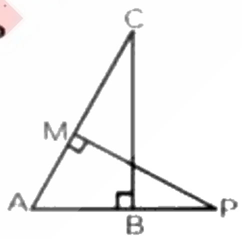
∴ By AA similarity

$\Delta ABE \sim \Delta CFB$

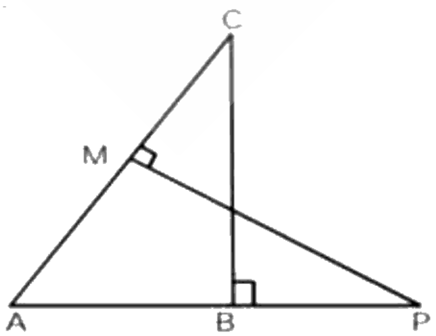
Q9. In figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:

(i) $\Delta ABC \sim \Delta AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$



Sol.



(i) In ΔABC and ΔAMP

$\angle CAB = \angle PAM$ (common)

$\angle ABC = \angle AMP = 90°$

$\therefore$ By AA similarity

$\Delta ABC \sim \Delta AMP$

(ii) As $\Delta ABC \sim \Delta AMP$ (Proved above)

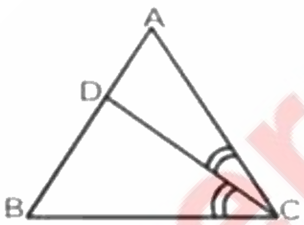
$$\therefore \quad \frac{CA}{PA} = \frac{BC}{MP}$$

Q10. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If $\Delta ABC \sim \Delta FEG$, show that :

(i) $\frac{CD}{GH} = \frac{AC}{FG}$ (ii) $\Delta DCB \sim \Delta HGE$

(iii) $\Delta DCA \sim \Delta HGF$

Sol. $\Delta ABC \sim \Delta FEG$



$\Rightarrow \angle ACB = \angle EGF$

$\Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle EGF$

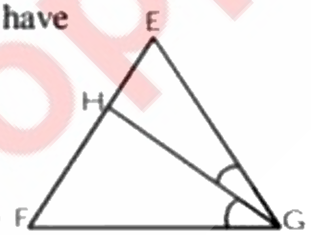
$\Rightarrow \angle DCB = \angle HGE$ ...(1)

Also, $\angle B = \angle E$

$\Rightarrow \angle DBC = \angle HEG$ ...(2)

From (1) and (2), we have

$\Rightarrow \Delta DCB \sim \Delta HGE$



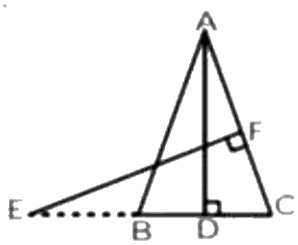
Similarly, we have

$\Delta DCA \sim \Delta HGF$

Now, $\Delta DCA \sim \Delta HGF$

$\Rightarrow \frac{DC}{HG} = \frac{CA}{GF} \Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$

Q11. In figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD $\perp$ BC and EF $\perp$ AC, prove that $\Delta ABD \sim \Delta ECF$.



Sol. In figure,

We are given that ΔABC is isosceles.

and $\quad AB = AC$

$\Rightarrow \quad \angle B = \angle C$ ...(1)

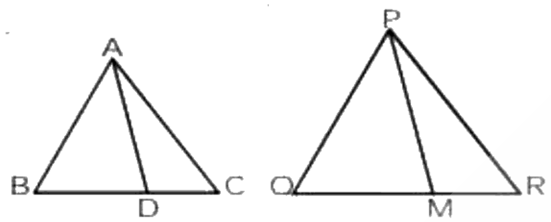
For triangles ABD and ECF,

$\angle ABD = \angle ECF \quad$ {from (1)}

and $\quad \angle ADB = \angle EFC \quad$ {each = 90°}

$\Rightarrow \quad \Delta ABD \sim \Delta ECF$ (AA similarity)

Q12. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.



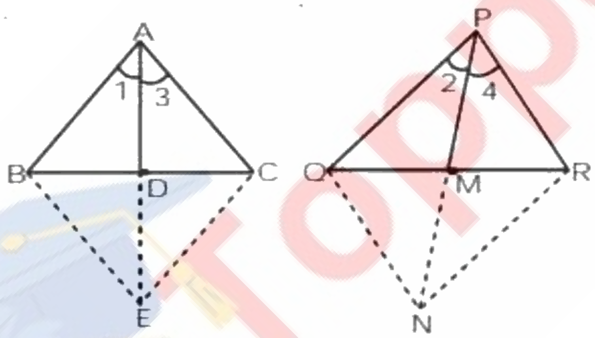
Sol. Given. ΔABC and ΔPQR. AD and PM are their medians respectively.

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \qquad ...(1)$$

To prove. $\Delta ABC \sim \Delta PQR$.

Construction : Produce AD to E such that AD = DE and produce PM to N such that PM = MN.

Join BE, CE, QN, RN.



Proof : Quadrilaterals ABEC and PQNR are parallelograms because their diagonals bisect each other at D and M respectively.

$\Rightarrow$ BE = AC and QN = PR.

$$\Rightarrow \frac{BE}{QN} = \frac{AC}{PR} \Rightarrow \frac{BE}{QN} = \frac{AB}{PQ} \quad \text{(By 1)}$$

i.e., $\frac{AB}{PQ} = \frac{BE}{QN}$ ...(2)

From (1), $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2AD}{2PM} = \frac{AE}{PN}$

i.e., $\frac{AB}{PQ} = \frac{AE}{PN}$ ...(3)

From (2) and (3), we have

$$\frac{AB}{PQ} = \frac{BE}{QN} = \frac{AE}{PN}$$

$\Rightarrow$ $\Delta ABE \sim \Delta PQN \Rightarrow \angle 1 = \angle 2$ ...(4)

Similarly, we can prove

$\Rightarrow$ $\Delta ACE \sim \Delta PRN \Rightarrow \angle 3 = \angle 4$ ...(5)

Adding (4) and (5), we have

$\Rightarrow$ $\angle 1 + \angle 3 = \angle 2 + \angle 4 \Rightarrow \angle A = \angle P$

$\Rightarrow$ $\Delta ABC \sim \Delta PQR$ (SAS similarity criterion)

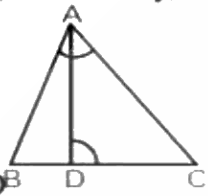
Q13. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB. CD$.

Sol. For ΔABC and ΔDAC, We have

$\angle BAC = \angle ADC$ (Given)

and $\angle ACB = \angle DCA$ (Each $= \angle C$)

$\Rightarrow$ $\Delta ABC \sim \Delta DAC$ (AA similarity)



$\Rightarrow$ $\frac{AC}{DC} = \frac{CB}{CA}$

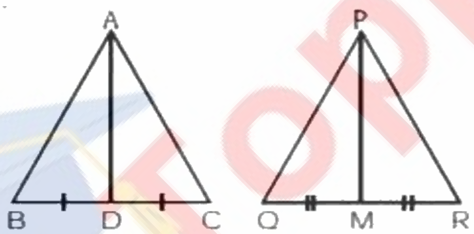
$\Rightarrow$ $\frac{CA}{CD} = \frac{CB}{CA}$

$\Rightarrow$ $CA \times CA = CB \times CD$

$\Rightarrow$ $CA^2 = CB \times CD$

Q14. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see figure). Show that $\Delta ABC \sim \Delta PQR$.

Sol.



As, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ (Given)

So, $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

$$\left\{\because \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{BD}{QM}\right\}$$

$\therefore$ By SSS similarity,

$\Delta ABD \sim \Delta PQM$.

As, $\Delta ABD \sim \Delta PQM$.

$\therefore$ $\angle ABD = \angle PQM$

Now, In ΔABC and ΔPQR

$$\frac{AB}{PQ} = \frac{BC}{QR} \text{ (Given)}$$

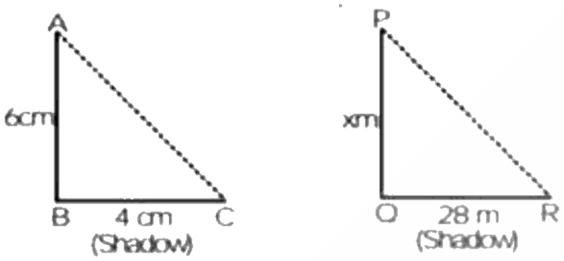
$\angle ABC = \angle PQR$ (Proved above)

$\therefore$ By SAS similarity

$\Delta ABC \sim \Delta PQR$.

Q15. A vertical stick of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Sol.



$\Delta ABC \sim \Delta PQR$

$$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR}$$

$$\frac{6}{x} = \frac{4}{28}$$

$\Rightarrow$ $x = 42$ m

Q16. If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Sol. $\Delta ABC \sim \Delta PQR$ (Given)

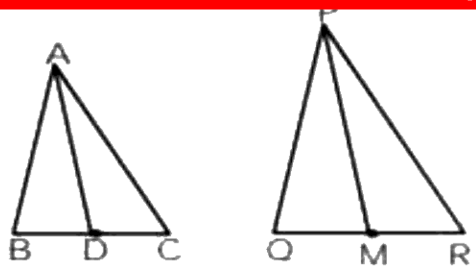
$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR};$$

$\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ ...(1)

Now, $BD = CD = \frac{1}{2}BC$

and $QM = RM = \frac{1}{2}QR$ ...(2)

($\because$ D is mid-point of BC and M is mid-point of QR)



From (1), $\frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM}$ (By (2))

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM}$

Thus, we have $\frac{AB}{PQ} = \frac{BD}{QM}$

and $\angle ABD = \angle PQM$ $(\because \angle B = \angle Q)$

$\Rightarrow \Delta ABD \sim \Delta PQM$ (By SAS similarity criterion)

$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$