Exercise 5.3

Question 1:

Find the sum of the following APs.

(i) 2, 7, 12 ,…., to 10 terms.

(ii) − 37, − 33, − 29 ,…, to 12 terms

(iii) 0.6, 1.7, 2.8 ,…….., to 100 terms

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$ **,………, to 11 terms**

Answer:

(i)2, 7, 12 ,…, to 10 terms

For this A.P.,

a = 2

d = $a_2 - a_1$ = 7 − 2 = 5

n = 10

We know that,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{10} = \frac{10}{2}\left[2(2) + (10-1)5\right]$$
$$= 5\left[4 + (9)\times(5)\right]$$
$$= 5 \times 49 = 245$$

(ii)−37, −33, −29 ,…, to 12 terms

For this A.P.,

a = −37

$d = a_2 - a_1 = (-33) - (-37)$

$= -33 + 37 = 4$

$n = 12$

We know that,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{12} = \frac{12}{2}\left[2(-37) + (12-1)4\right]$$
$$= 6\left[-74 + 11 \times 4\right]$$
$$= 6\left[-74 + 44\right]$$
$$= 6(-30) = -180$$

(iii) 0.6, 1.7, 2.8 ,..., to 100 terms

For this A.P.,

$a = 0.6$

$d = a_2 - a_1 = 1.7 - 0.6 = 1.1$

$n = 100$

We know that,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{100} = \frac{100}{2}\left[2(0.6) + (100-1)1.1\right]$$
$$= 50\left[1.2 + (99) \times (1.1)\right]$$
$$= 50\left[1.2 + 108.9\right]$$
$$= 50\left[110.1\right]$$
$$= 5505$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10},$ . ... , to 11 terms

For this A.P.,

$$a = \frac{1}{15}$$

n = 11

$$\begin{aligned} d = a_2 - a_1 &= \frac{1}{12} - \frac{1}{15} \\ &= \frac{5-4}{60} = \frac{1}{60} \end{aligned}$$

We know that,

$$\begin{aligned} S_n &= \frac{n}{2}\left[2a + (n-1)d\right] \\ S_{11} &= \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\frac{1}{60}\right] \\ &= \frac{11}{2}\left[\frac{2}{15} + \frac{10}{60}\right] \\ &= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right] = \frac{11}{2}\left[\frac{4+5}{30}\right] \\ &= \left(\frac{11}{2}\right)\left(\frac{9}{30}\right) = \frac{33}{20} \end{aligned}$$

Question 2:

Find the sums given below

(i) 7 + $10\frac{1}{2}$ + 14 + ………… + 84

(ii) 34 + 32 + 30 + ……….. + 10

(iii) − 5 + (− 8) + (− 11) + ………… + (− 230)

Answer:

(i)7 + $10\frac{1}{2}$ + 14 + …………+ 84

For this A.P.,

a = 7

l = 84

$$d = a_2 - a_1 = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$$

Let 84 be the n^{th} term of this A.P.

l = a + (n − 1)d

$$84 = 7 + (n-1)\frac{7}{2}$$

$$77 = (n-1)\frac{7}{2}$$

22 = n − 1

n = 23

We know that,

$$S_n = \frac{n}{2}(a+l)$$

$$S_n = \frac{23}{2}[7+84]$$

$$= \frac{23 \times 91}{2} = \frac{2093}{2}$$

$$= 1046\frac{1}{2}$$

(ii)34 + 32 + 30 + ……….. + 10

For this A.P.,

a = 34

$d = a_2 - a_1 = 32 - 34 = -2$

l = 10

Let 10 be the n^{th} term of this A.P.

l = a + (n − 1) d

10 = 34 + (n − 1) (−2)

−24 = (n − 1) (−2)

12 = n − 1

n = 13

$$S_n = \frac{n}{2}(a+l)$$

$$= \frac{13}{2}(34+10)$$
$$= \frac{13 \times 44}{2} = 13 \times 22$$
$$= 286$$

(iii)(−5) + (−8) + (−11) + ………… + (−230)

For this A.P.,

a = −5

l = −230

$d = a_2 - a_1 = (-8) - (-5)$

= − 8 + 5 = −3

Let −230 be the n^{th} term of this A.P.

l = a + (n − 1)d

−230 = − 5 + (n − 1) (−3)

$-225 = (n - 1)(-3)$

$(n - 1) = 75$

$n = 76$

And, $S_n = \frac{n}{2}(a + l)$

$$= \frac{76}{2}[(-5) + (-230)]$$
$$= 38(-235)$$
$$= -8930$$

Question 3:

In an AP

(i) Given a = 5, d = 3, a_n = 50, find n and S_n.

(ii) Given a = 7, a_{13} = 35, find d and S_{13}.

(iii) Given a_{12} = 37, d = 3, find a and S_{12}.

(iv) Given a_3 = 15, S_{10} = 125, find d and a_{10}.

(v) Given d = 5, S_9 = 75, find a and a_9.

(vi) Given a = 2, d = 8, S_n = 90, find n and a_n.

(vii) Given a = 8, a_n = 62, S_n = 210, find n and d.

(viii) Given a_n = 4, d = 2, S_n = − 14, find n and a.

(ix) Given a = 3, n = 8, S = 192, find d.

(x)Given l = 28, S = 144 and there are total 9 terms. Find a.

Answer:

(i) Given that, a = 5, d = 3, a_n = 50

As $a_n = a + (n - 1)d$,

$\therefore 50 = 5 + (n - 1)3$

45 = (n − 1)3

15 = n − 1

n = 16

$$S_n = \frac{n}{2}[a + a_n]$$
$$S_{16} = \frac{16}{2}[5 + 50]$$
$$= 8 \times 55$$
$$= 440$$

(ii) Given that, a = 7, a_{13} = 35

As $a_n = a + (n - 1)d$,

∴ $a_{13} = a + (13 - 1)d$

35 = 7 + 12 d

35 − 7 = 12d

28 = 12d

$$d = \frac{7}{3}$$

$$S_n = \frac{n}{2}[a + a_n]$$
$$S_{13} = \frac{n}{2}[a + a_{13}]$$
$$= \frac{13}{2}[7 + 35]$$
$$= \frac{13 \times 42}{2} = 13 \times 21$$
$$= 273$$

(iii)Given that, a_{12} = 37, d = 3

As $a_n = a + (n - 1)d$,

$a_{12} = a + (12 - 1)3$

$37 = a + 33$

$a = 4$

$$S_n = \frac{n}{2}[a + a_n]$$
$$S_n = \frac{12}{2}[4 + 37]$$
$$S_n = 6(41)$$
$$S_n = 246$$

(iv) Given that, $a_3 = 15$, $S_{10} = 125$

As $a_n = a + (n - 1)d$,

$a_3 = a + (3 - 1)d$

$15 = a + 2d$ (i)

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{10} = \frac{10}{2}[2a + (10-1)d]$$
$$125 = 5(2a + 9d)$$
$$25 = 2a + 9d \qquad \text{(ii)}$$

On multiplying equation (1) by 2, we obtain

$30 = 2a + 4d$ (iii)

On subtracting equation (iii) from (ii), we obtain

$-5 = 5d$

d = −1

From equation (i),

15 = a + 2(−1)

15 = a − 2

a = 17

a_{10} = a + (10 − 1)d

a_{10} = 17 + (9) (−1)

a_{10} = 17 − 9 = 8

(v)Given that, d = 5, S_9 = 75

As $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$,

$$S_9 = \frac{9}{2}\left[2a+(9-1)5\right]$$
$$75 = \frac{9}{2}(2a+40)$$

25 = 3(a + 20)

25 = 3a + 60

3a = 25 − 60

$$a = \frac{-35}{3}$$

a_n = a + (n − 1)d

a_9 = a + (9 − 1) (5)

$$= \frac{-35}{3} + 8(5)$$

$$= \frac{-35}{3} + 40$$

$$= \frac{-35+120}{3} = \frac{85}{3}$$

(vi) Given that, a = 2, d = 8, S_n = 90

As $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$,

$$90 = \frac{n}{2}\left[4 + (n-1)8\right]$$

90 = n [2 + (n − 1)4]

90 = n [2 + 4n − 4]

90 = n (4n − 2) = $4n^2$ − 2n

$4n^2$ − 2n − 90 = 0

$4n^2$ − 20n + 18n − 90 = 0

4n (n − 5) + 18 (n − 5) = 0

(n − 5) (4n + 18) = 0

Either n − 5 = 0 or 4n + 18 = 0

n = 5 or $n = -\frac{18}{4} = \frac{-9}{2}$

However, n can neither be negative nor fractional.

Therefore, n = 5

$a_n = a + (n - 1)d$

$a_5 = 2 + (5 - 1)8$

$= 2 + (4)(8)$

$= 2 + 32 = 34$

(vii) Given that, $a = 8$, $a_n = 62$, $S_n = 210$

$$S_n = \frac{n}{2}[a + a_n]$$
$$210 = \frac{n}{2}[8 + 62]$$
$$210 = \frac{n}{2}(70)$$

$n = 6$

$a_n = a + (n - 1)d$

$62 = 8 + (6 - 1)d$

$62 - 8 = 5d$

$54 = 5d$

$$d = \frac{54}{5}$$

(viii) Given that, $a_n = 4$, $d = 2$, $S_n = -14$

$a_n = a + (n - 1)d$

$4 = a + (n - 1)2$

$4 = a + 2n - 2$

a + 2n = 6

a = 6 − 2n (i)

$$S_n = \frac{n}{2}[a + a_n]$$

$$-14 = \frac{n}{2}[a + 4]$$

−28 = n (a + 4)

−28 = n (6 − 2n + 4) {From equation (i)}

−28 = n (− 2n + 10)

$-28 = -2n^2 + 10n$

$2n^2 - 10n - 28 = 0$

$n^2 - 5n - 14 = 0$

$n^2 - 7n + 2n - 14 = 0$

n (n − 7) + 2(n − 7) = 0

(n − 7) (n + 2) = 0

Either n − 7 = 0 or n + 2 = 0

n = 7 or n = −2

However, n can neither be negative nor fractional.

Therefore, n = 7

From equation (i), we obtain

a = 6 − 2n

a = 6 − 2(7)

= 6 − 14

= −8

(ix)Given that, a = 3, n = 8, S = 192

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$192 = \frac{8}{2}\left[2\times 3 + (8-1)d\right]$$

192 = 4 [6 + 7d]

48 = 6 + 7d

42 = 7d

d = 6

(x)Given that, l = 28, S = 144 and there are total of 9 terms.

$$S_n = \frac{n}{2}(a + l)$$
$$144 = \frac{9}{2}(a + 28)$$

(16) × (2) = a + 28

32 = a + 28

a = 4

Question 4:

How many terms of the AP. 9, 17, 25 … must be taken to give a sum of 636?

Answer:

Let there be n terms of this A.P.

For this A.P., a = 9

$d = a_2 - a_1 = 17 - 9 = 8$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$636 = \frac{n}{2}\left[2 \times a + (n-1)8\right]$$
$$636 = \frac{n}{2}\left[18 + (n-1)8\right]$$

636 = n [9 + 4n − 4]

636 = n (4n + 5)

$4n^2 + 5n - 636 = 0$

$4n^2 + 53n - 48n - 636 = 0$

n (4n + 53) − 12 (4n + 53) = 0

(4n + 53) (n − 12) = 0

Either 4n + 53 = 0 or n − 12 = 0

$n = \frac{-53}{4}$ or n = 12

n cannot be $-\frac{53}{4}$. As the number of terms can neither be negative nor fractional, therefore, n = 12 only.

Question 5:

The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Answer:

Given that,

a = 5

l = 45

S_n = 400

$$S_n = \frac{n}{2}(a+l)$$
$$400 = \frac{n}{2}(5+45)$$
$$400 = \frac{n}{2}(50)$$

n = 16

l = a + (n − 1) d

45 = 5 + (16 − 1) d

40 = 15d

$$d = \frac{40}{15} = \frac{8}{3}$$

Question 6:

The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Answer:

Given that,

a = 17

l = 350

d = 9

Let there be n terms in the A.P.

l = a + (n − 1) d

350 = 17 + (n − 1)9

333 = (n − 1)9

(n − 1) = 37

n = 38

$$S_n = \frac{n}{2}(a + l)$$
$$\Rightarrow S_n = \frac{38}{2}(17 + 350) = 19(367) = 6973$$

Thus, this A.P. contains 38 terms and the sum of the terms of this A.P. is 6973.

Question 7:

Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

Answer:

d = 7

a_{22} = 149

S_{22} = ?

a_n = a + (n − 1)d

a_{22} = a + (22 − 1)d

149 = a + 21 × 7

149 = a + 147

a = 2

$$S_n = \frac{n}{2}(a + a_n)$$
$$= \frac{22}{2}(2 + 149)$$
$$= 11(151) = 1661$$

Question 8:

Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Answer:

Given that,

$a_2 = 14$

$a_3 = 18$

$d = a_3 - a_2 = 18 - 14 = 4$

$a_2 = a + d$

$14 = a + 4$

$a = 10$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{51} = \frac{51}{2}[2 \times 10 + (51-1)4]$$
$$= \frac{51}{2}[20 + (50)(4)]$$
$$= \frac{51(220)}{2} = 51(110)$$

$= 5610$

Question 9:

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Answer:

Given that,

$S_7 = 49$

$S_{17} = 289$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_7 = \frac{7}{2}\left[2a + (7-1)d\right]$$
$$49 = \frac{7}{2}(2a + 6d)$$

7 = (a + 3d)

a + 3d = 7 (i)

Similarly, $S_{17} = \frac{17}{2}\left[2a + (17-1)d\right]$

$$289 = \frac{17}{2}\left[2a + 16d\right]$$

17 = (a + 8d)

a + 8d = 17 (ii)

Subtracting equation (i) from equation (ii),

5d = 10

d = 2

From equation (i),

a + 3(2) = 7

a + 6 = 7

a = 1

$$\begin{aligned} S_n &= \frac{n}{2}\left[2a+(n-1)d\right] \\ &= \frac{n}{2}\left[2(1)+(n-1)(2)\right] \\ &= \frac{n}{2}(2+2n-2) \\ &= \frac{n}{2}(2n) \end{aligned}$$

$= n^2$

Question 10:

Show that $a_1, a_2 \dots, a_n, \dots$ form an AP where a_n is defined as below

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Answer:

(i) $a_n = 3 + 4n$

$a_1 = 3 + 4(1) = 7$

$a_2 = 3 + 4(2) = 3 + 8 = 11$

$a_3 = 3 + 4(3) = 3 + 12 = 15$

$a_4 = 3 + 4(4) = 3 + 16 = 19$

It can be observed that

$a_2 - a_1 = 11 - 7 = 4$

$a_3 - a_2 = 15 - 11 = 4$

$a_4 - a_3 = 19 - 15 = 4$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, this is an AP with common difference as 4 and first term as 7.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{15} = \frac{15}{2}\left[2(7) + (15-1)4\right]$$
$$= \frac{15}{2}\left[(14) + 56\right]$$
$$= \frac{15}{2}(70)$$

$= 15 \times 35$

$= 525$

(ii) $a_n = 9 - 5n$

$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$

$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$

$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$

$a_4 = 9 - 5 \times 4 = 9 - 20 = -11$

It can be observed that

$a_2 - a_1 = -1 - 4 = -5$

$a_3 - a_2 = -6 - (-1) = -5$

$a_4 - a_3 = -11 - (-6) = -5$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, this is an A.P. with common difference as −5 and first term as 4.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{15} = \frac{15}{2}\left[2(4) + (15-1)(-5)\right]$$
$$= \frac{15}{2}\left[8 + 14(-5)\right]$$
$$= \frac{15}{2}(8-70)$$
$$= \frac{15}{2}(-62) = 15(-31)$$

= −465

Question 11:

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3^{rd}, the10^{th} and the n^{th} terms.

Answer:

Given that,

$S_n = 4n - n^2$

First term, $a = S_1 = 4(1) - (1)^2 = 4 - 1 = 3$

Sum of first two terms = S_2

$= 4(2) - (2)^2 = 8 - 4 = 4$

Second term, $a_2 = S_2 - S_1 = 4 - 3 = 1$

$d = a_2 - a = 1 - 3 = -2$

$a_n = a + (n - 1)d$

$= 3 + (n - 1)(-2)$

$= 3 - 2n + 2$

$= 5 - 2n$

Therefore, $a_3 = 5 - 2(3) = 5 - 6 = -1$

$a_{10} = 5 - 2(10) = 5 - 20 = -15$

Hence, the sum of first two terms is 4. The second term is 1. 3^{rd}, 10^{th}, and n^{th} terms are −1, −15, and 5 − 2n respectively.

Question 12:

Find the sum of first 40 positive integers divisible by 6.

Answer:

The positive integers that are divisible by 6 are

6, 12, 18, 24 …

It can be observed that these are making an A.P. whose first term is 6 and common difference is 6.

$a = 6$

$d = 6$

$S_{40} = ?$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{40} = \frac{40}{2}\left[2(6) + (40-1)6\right]$$

= 20[12 + (39) (6)]

= 20(12 + 234)

= 20 × 246

= 4920

Question 13:

Find the sum of first 15 multiples of 8.

Answer:

The multiples of 8 are

8, 16, 24, 32...

These are in an A.P., having first term as 8 and common difference as 8.

Therefore, a = 8

d = 8

S_{15} =?

$$\begin{aligned} S_n &= \frac{n}{2}\left[2a + (n-1)d\right] \\ &= \frac{15}{2}\left[2(8) + (15-1)8\right] \\ &= \frac{15}{2}\left[16 + 14(8)\right] \\ &= \frac{15}{2}(16 + 112) \\ &= \frac{15(128)}{2} = 15 \times 64 \end{aligned}$$

= 960

Question 14:

Find the sum of the odd numbers between 0 and 50.

Answer:

The odd numbers between 0 and 50 are

1, 3, 5, 7, 9 … 49

Therefore, it can be observed that these odd numbers are in an A.P.

a = 1

d = 2

l = 49

l = a + (n − 1) d

49 = 1 + (n − 1)2

48 = 2(n − 1)

n − 1 = 24

n = 25

$$S_n = \frac{n}{2}(a+l)$$

$$S_{25} = \frac{25}{2}(1+49)$$
$$= \frac{25(50)}{2} = (25)(25)$$

= 625

Question 15:

A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

Answer:

It can be observed that these penalties are in an A.P. having first term as 200 and common difference as 50.

a = 200

d = 50

Penalty that has to be paid if he has delayed the work by 30 days = S_{30}

$$=\frac{30}{2}\left[2(200)+(30-1)50\right]$$

= 15 [400 + 1450]

= 15 (1850)

= 27750

Therefore, the contractor has to pay Rs 27750 as penalty.

Question 16:

A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Answer:

Let the cost of 1^{st} prize be P.

Cost of 2^{nd} prize = P − 20

And cost of 3^{rd} prize = P − 40

It can be observed that the cost of these prizes are in an A.P. having common difference as −20 and first term as P.

a = P

d = −20

Given that, S_7 = 700

$$\frac{7}{2}\left[2a+(7-1)d\right]=700$$
$$\frac{\left[2a+(6)(-20)\right]}{2}=100$$

a + 3(−20) = 100

a − 60 = 100

a = 160

Therefore, the value of each of the prizes was Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.

Question 17:

In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Answer:

It can be observed that the number of trees planted by the students is in an AP.

1, 2, 3, 4, 5………………..12

First term, a = 1

Common difference, d = 2 − 1 = 1

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{12} = \frac{12}{2}\left[2(1) + (12-1)(1)\right]$$

= 6 (2 + 11)

= 6 (13)

= 78

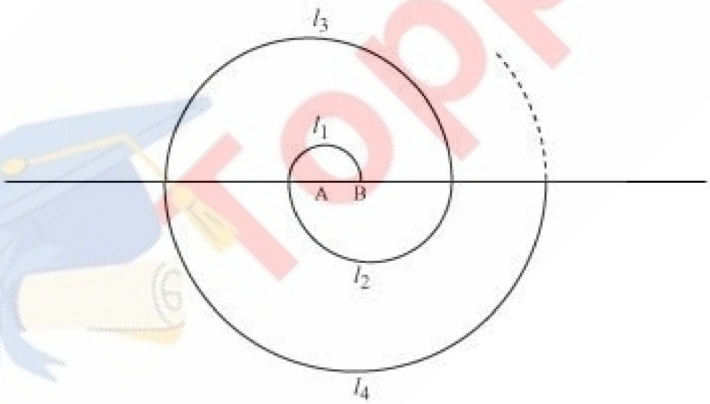
Therefore, number of trees planted by 1 section of the classes = 78

Number of trees planted by 3 sections of the classes = 3 × 78 = 234

Therefore, 234 trees will be planted by the students.

Question 18:

A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A of radii 0.5, 1.0 cm, 1.5 cm, 2.0 cm, ……… as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? $\left(\text{Take } \pi = \frac{22}{7}\right)$



Answer:

Semi-perimeter of circle = πr

$l_1 = \pi(0.5) = \frac{\pi}{2}$ cm

$l_2 = \pi(1) = \pi$ cm

$l_3 = \pi(1.5) = \frac{3\pi}{2}$ cm

Therefore, l_1, l_2, l_3 ,i.e. the lengths of the semi-circles are in an A.P.,

$$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots\ldots\ldots\ldots$$

$$a = \frac{\pi}{2}$$
$$d = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

S_{13} =?

We know that the sum of n terms of an a A.P. is given by

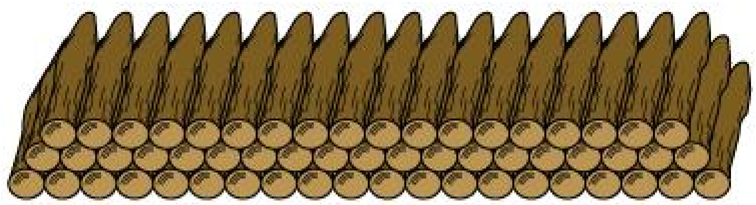
$$\begin{aligned} S_n &= \frac{n}{2}\left[2a + (n-1)d\right] \\ &= \frac{13}{2}\left[2\left(\frac{\pi}{2}\right) + (13-1)\left(\frac{\pi}{2}\right)\right] \\ &= \frac{13}{2}\left[\pi + \frac{12\pi}{2}\right] \\ &= \left(\frac{13}{2}\right)(7\pi) \\ &= \frac{91\pi}{2} \\ &= \frac{91 \times 22}{2 \times 7} = 13 \times 11 \end{aligned}$$

= 143

Therefore, the length of such spiral of thirteen consecutive semi-circles will be 143 cm.

Question 19:

200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?



Answer:

It can be observed that the numbers of logs in rows are in an A.P.

20, 19, 18…

For this A.P.,

$a = 20$

$d = a_2 - a_1 = 19 - 20 = -1$

Let a total of 200 logs be placed in n rows.

$S_n = 200$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$200 = \frac{n}{2}\left[2(20) + (n-1)(-1)\right]$$

$400 = n (40 - n + 1)$

$400 = n (41 - n)$

$400 = 41n - n^2$

$n^2 - 41n + 400 = 0$

$n^2 - 16n - 25n + 400 = 0$

$n(n-16) - 25(n-16) = 0$

$(n-16)(n-25) = 0$

Either $(n-16) = 0$ or $n - 25 = 0$

$n = 16$ or $n = 25$

$a_n = a + (n-1)d$

$a_{16} = 20 + (16-1)(-1)$

$a_{16} = 20 - 15$

$a_{16} = 5$

Similarly,

$a_{25} = 20 + (25-1)(-1)$

$a_{25} = 20 - 24$

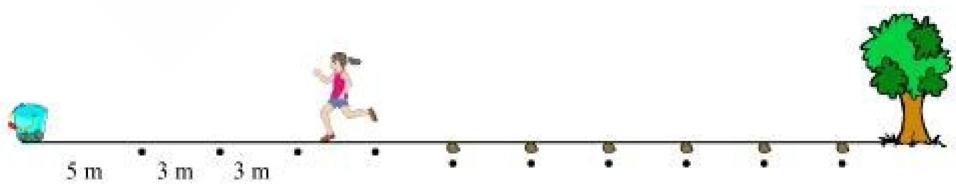
$= -4$

Clearly, the number of logs in 16^{th} row is 5. However, the number of logs in 25^{th} row is negative, which is not possible.

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16^{th} row is 5.

Question 20:

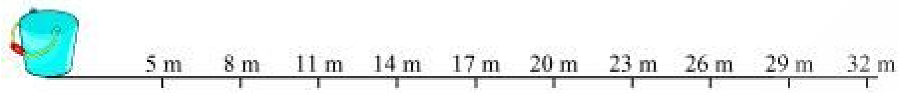
In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line.



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[Hint: to pick up the first potato and the second potato, the total distance (in metres) run by a competitor is 2 × 5 + 2 ×(5 + 3)]

Answer:



The distances of potatoes are as follows.

5, 8, 11, 14…

It can be observed that these distances are in A.P.

a = 5

d = 8 − 5 = 3

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2(5) + (10-1)3\right]$$

= 5[10 + 9 × 3]

= 5(10 + 27) = 5(37)

= 185

As every time she has to run back to the bucket, therefore, the total distance that the competitor has to run will be two times of it.

Therefore, total distance that the competitor will run = 2 × 185

= 370 m

Alternatively,

The distances of potatoes from the bucket are 5, 8, 11, 14…

Distance run by the competitor for collecting these potatoes are two times of the distance at which the potatoes have been kept. Therefore, distances to be run are

10, 16, 22, 28, 34,……….

a = 10

d = 16 − 10 = 6

S_{10} =?

$$S_{10} = \frac{10}{2}\left[2\times 10 + (10-1)6\right]$$

= 5[20 + 54]

= 5 (74)

= 370

Therefore, the competitor will run a total distance of 370 m.