Exercise 5.2

Question 1:

Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n^{th} term of the A.P.

	a	d	n	a_n
I	7	3	8	……
II	− 18	…..	10	0
III	…..	− 3	18	− 5
IV	− 18.9	2.5	…..	3.6
V	3.5	0	105	…..

Answer:

I. $a = 7, d = 3, n = 8, a_n = ?$

We know that,

For an A.P. $a_n = a + (n - 1) d$

$= 7 + (8 - 1) 3$

$= 7 + (7) 3$

$= 7 + 21 = 28$

Hence, $a_n = 28$

II. Given that

$a = -18, n = 10, a_n = 0, d = ?$

We know that,

$a_n = a + (n - 1)\, d$

$0 = -18 + (10 - 1)\, d$

$18 = 9d$

$$d = \frac{18}{9} = 2$$

Hence, common difference, d = 2

III. Given that

$d = -3$, $n = 18$, $a_n = -5$

We know that,

$a_n = a + (n - 1)\, d$

$-5 = a + (18 - 1)(-3)$

$-5 = a + (17)(-3)$

$-5 = a - 51$

$a = 51 - 5 = 46$

Hence, a = 46

IV. $a = -18.9$, $d = 2.5$, $a_n = 3.6$, $n = ?$

We know that,

$a_n = a + (n - 1)\, d$

$3.6 = -18.9 + (n - 1)\, 2.5$

$3.6 + 18.9 = (n - 1)\, 2.5$

22.5 = (n − 1) 2.5

$$(n-1)=\frac{22.5}{2.5}$$
$$n-1=9$$
$$n=10$$

Hence, n = 10

V. a = 3.5, d = 0, n = 105, a_n = ?

We know that,

a_n = a + (n − 1) d

a_n = 3.5 + (105 − 1) 0

a_n = 3.5 + 104 × 0

a_n = 3.5

Hence, a_n = 3.5

Question 2:

Choose the correct choice in the following and justify

I. 30th term of the A.P: 10, 7, 4, …, is

A. 97 B. 77 C. − 77 D. − 87

II 11th term of the A.P. $-3,-\frac{1}{2},2,\ldots$ **is**

A. 28 B. 22 C. − 38 D. $-48\frac{1}{2}$

Answer:

I. Given that

A.P. 10, 7, 4, …

First term, a = 10

Common difference, $d = a_2 - a_1 = 7 - 10$

$= -3$

We know that, $a_n = a + (n - 1) d$

$a_{30} = 10 + (30 - 1) (-3)$

$a_{30} = 10 + (29) (-3)$

$a_{30} = 10 - 87 = -77$

Hence, the correct answer is C.

II. Given that, A.P. $-3, -\frac{1}{2}, 2, \ldots$

First term a = −3

Common difference, $d = a_2 - a_1$

$$= -\frac{1}{2} - (-3)$$

$$= -\frac{1}{2} + 3 = \frac{5}{2}$$

We know that,

$$a_n = a + (n-1)d$$

$$a_{11} = -3 + (11-1)\left(\frac{5}{2}\right)$$

$$a_{11} = -3 + (10)\left(\frac{5}{2}\right)$$

$$a_{11} = -3 + 25$$

$$a_{11} = 22$$

Hence, the answer is B.

Question 3:

In the following APs find the missing term in the boxes

I. $2, \square, 26$

II. $\square, 13, \square, 3$

III. $5, \square, \square, 9\frac{1}{2}$

IV. $-4, \square, \square, \square, \square, 6$

V. $\square, 38, \square, \square, \square, -22$

Answer:

I. $2, \square, 26$

For this A.P.,

$a = 2$

$a_3 = 26$

We know that, $a_n = a + (n - 1) d$

$a_3 = 2 + (3 - 1) d$

26 = 2 + 2d

24 = 2d

d = 12

$a_2 = 2 + (2 - 1)\ 12$

= 14

Therefore, 14 is the missing term.

II. $\square$, 13, $\square$, 3

For this A.P.,

$a_2 = 13$ and

$a_4 = 3$

We know that, $a_n = a + (n - 1)\ d$

$a_2 = a + (2 - 1)\ d$

13 = a + d (I)

$a_4 = a + (4 - 1)\ d$

3 = a + 3d (II)

On subtracting (I) from (II), we obtain

−10 = 2d

d = −5

From equation (I), we obtain

13 = a + (−5)

a = 18

a_3 = 18 + (3 − 1) (−5)

= 18 + 2 (−5) = 18 − 10 = 8

Therefore, the missing terms are 18 and 8 respectively.

III. $5, \square, \square, 9\frac{1}{2}$

For this A.P.,

$$a = 5$$
$$a_4 = 9\frac{1}{2} = \frac{19}{2}$$

We know that,

$$a_n = a + (n-1)d$$
$$a_4 = a + (4-1)d$$
$$\frac{19}{2} = 5 + 3d$$
$$\frac{19}{2} - 5 = 3d$$
$$\frac{9}{2} = 3d$$
$$d = \frac{3}{2}$$
$$a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$$
$$a_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = 8$$

Therefore, the missing terms are $\frac{13}{2}$ and 8 respectively.

IV. $-4, \square, \square, \square, \square, 6$

For this A.P.,

$a = -4$ and

$a_6 = 6$

We know that,

$a_n = a + (n - 1) d$

$a_6 = a + (6 - 1) d$

$6 = -4 + 5d$

$10 = 5d$

$d = 2$

$a_2 = a + d = -4 + 2 = -2$

$a_3 = a + 2d = -4 + 2(2) = 0$

$a_4 = a + 3d = -4 + 3(2) = 2$

$a_5 = a + 4d = -4 + 4(2) = 4$

Therefore, the missing terms are −2, 0, 2, and 4 respectively.

V. $\square, 38, \square, \square, \square, -22$

For this A.P.,

$a_2 = 38$

$a_6 = -22$

We know that

$a_n = a + (n - 1) d$

$a_2 = a + (2 - 1) d$

38 = a + d (1)

$a_6 = a + (6 - 1) d$

−22 = a + 5d (2)

On subtracting equation (1) from (2), we obtain

− 22 − 38 = 4d

−60 = 4d

d = −15

$a = a_2 - d = 38 - (-15) = 53$

$a_3 = a + 2d = 53 + 2 (-15) = 23$

$a_4 = a + 3d = 53 + 3 (-15) = 8$

$a_5 = a + 4d = 53 + 4 (-15) = -7$

Therefore, the missing terms are 53, 23, 8, and −7 respectively.

Question 4:

Which term of the A.P. 3, 8, 13, 18, … is 78?

Answer:

3, 8, 13, 18, …

For this A.P.,

a = 3

$d = a_2 - a_1 = 8 - 3 = 5$

Let n^{th} term of this A.P. be 78.

$a_n = a + (n - 1) d$

78 = 3 + (n − 1) 5

75 = (n − 1) 5

(n − 1) = 15

n = 16

Hence, 16^{th} term of this A.P. is 78.

Question 5:

Find the number of terms in each of the following A.P.

I. 7, 13, 19, …, 205

II. $18, 15\frac{1}{2}, 13, \ldots, -47$

Answer:

I. 7, 13, 19, …, 205

For this A.P.,

a = 7

$d = a_2 - a_1 = 13 - 7 = 6$

Let there are n terms in this A.P.

$a_n = 205$

We know that

$a_n = a + (n - 1)d$

Therefore, 205 = 7 + (n − 1) 6

198 = (n − 1) 6

33 = (n − 1)

n = 34

Therefore, this given series has 34 terms in it.

II. $18,\ 15\frac{1}{2},\ 13, \ldots, -47$

For this A.P.,

$$a = 18$$

$$d = a_2 - a_1 = 15\frac{1}{2} - 18$$

$$d = \frac{31-36}{2} = -\frac{5}{2}$$

Let there are n terms in this A.P.

Therefore, $a_n = -47$ and we know that,

$$a_n = a + (n-1)d$$
$$-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$$
$$-47 - 18 = (n-1)\left(-\frac{5}{2}\right)$$
$$-65 = (n-1)\left(-\frac{5}{2}\right)$$
$$(n-1) = \frac{-130}{-5}$$
$$(n-1) = 26$$
$$n = 27$$

Therefore, this given A.P. has 27 terms in it.

Question 6:

Check whether − 150 is a term of the A.P. 11, 8, 5, 2, …

Answer:

For this A.P.,

a = 11

$d = a_2 - a_1 = 8 - 11 = -3$

Let −150 be the n^{th} term of this A.P.

We know that,

$$a_n = a + (n-1)d$$
$$-150 = 11 + (n-1)(-3)$$
$$-150 = 11 - 3n + 3$$
$$-164 = -3n$$
$$n = \frac{164}{3}$$

Clearly, n is not an integer.

Therefore, −150 is not a term of this A.P.

Question 7:

Find the 31st term of an A.P. whose 11th term is 38 and the 16th term is 73

Answer:

Given that,

$a_{11} = 38$

$a_{16} = 73$

We know that,

$a_n = a + (n - 1) d$

$a_{11} = a + (11 - 1) d$

$38 = a + 10d$ (1)

Similarly,

$a_{16} = a + (16 - 1) d$

$73 = a + 15d$ (2)

On subtracting (1) from (2), we obtain

$35 = 5d$

$d = 7$

From equation (1),

$38 = a + 10 \times (7)$

$38 - 70 = a$

$a = -32$

$a_{31} = a + (31 - 1) d$

$= - 32 + 30 (7)$

$= - 32 + 210$

$= 178$

Hence, 31st term is 178.

Question 8:

An A.P. consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term

Answer:

Given that,

$a_3 = 12$

$a_{50} = 106$

We know that,

$a_n = a + (n - 1) d$

$a_3 = a + (3 - 1) d$

$12 = a + 2d$ (I)

Similarly, $a_{50} = a + (50 - 1) d$

$106 = a + 49d$ (II)

On subtracting (I) from (II), we obtain

$94 = 47d$

$d = 2$

From equation (I), we obtain

$12 = a + 2\ (2)$

$a = 12 - 4 = 8$

$a_{29} = a + (29 - 1)\ d$

$a_{29} = 8 + (28)2$

$a_{29} = 8 + 56 = 64$

Therefore, 29th term is 64.

Question 9:

If the 3rd and the 9th terms of an A.P. are 4 and – 8 respectively. Which term of this A.P. is zero.

Answer:

Given that,

$a_3 = 4$

$a_9 = -8$

We know that,

$a_n = a + (n - 1)\ d$

$a_3 = a + (3 - 1)\ d$

$4 = a + 2d$ (I)

$a_9 = a + (9 - 1)\ d$

$-8 = a + 8d$ (II)

On subtracting equation (I) from (II), we obtain

$-12 = 6d$

$d = -2$

From equation (I), we obtain

$4 = a + 2(-2)$

$4 = a - 4$

$a = 8$

Let n^{th} term of this A.P. be zero.

$a_n = a + (n - 1) d$

$0 = 8 + (n - 1)(-2)$

$0 = 8 - 2n + 2$

$2n = 10$

$n = 5$

Hence, 5^{th} term of this A.P. is 0.

Question 10:

If 17^{th} term of an A.P. exceeds its 10^{th} term by 7. Find the common difference.

Answer:

We know that,

For an A.P., $a_n = a + (n - 1) d$

$a_{17} = a + (17 - 1) d$

$a_{17} = a + 16d$

Similarly, $a_{10} = a + 9d$

It is given that

$a_{17} - a_{10} = 7$

$(a + 16d) - (a + 9d) = 7$

$7d = 7$

$d = 1$

Therefore, the common difference is 1.

Question 11:

Which term of the A.P. 3, 15, 27, 39, … will be 132 more than its 54th term?

Answer:

Given A.P. is 3, 15, 27, 39, …

$a = 3$

$d = a_2 - a_1 = 15 - 3 = 12$

$a_{54} = a + (54 - 1) d$

$= 3 + (53) (12)$

$= 3 + 636 = 639$

$132 + 639 = 771$

We have to find the term of this A.P. which is 771.

Let n^{th} term be 771.

$a_n = a + (n - 1) d$

$771 = 3 + (n - 1) 12$

$768 = (n - 1) 12$

$(n - 1) = 64$

$n = 65$

Therefore, 65^{th} term was 132 more than 54^{th} term.

Alternatively,

Let n^{th} term be 132 more than 54^{th} term.

$$n = 54 + \frac{132}{12}$$
$$= 54 + 11 = 65^{th} \text{ term}$$

Question 12:

Two APs have the same common difference. The difference between their 100^{th} term is 100, what is the difference between their 1000^{th} terms?

Answer:

Let the first term of these A.P.s be a_1 and a_2 respectively and the common difference of these A.P.s be d.

For first A.P.,

$a_{100} = a_1 + (100 - 1) d$

$= a_1 + 99d$

$a_{1000} = a_1 + (1000 - 1) d$

$a_{1000} = a_1 + 999d$

For second A.P.,

$a_{100} = a_2 + (100 - 1)\ d$

$= a_2 + 99d$

$a_{1000} = a_2 + (1000 - 1)\ d$

$= a_2 + 999d$

Given that, difference between

100^{th} term of these A.P.s = 100

Therefore, $(a_1 + 99d) - (a_2 + 99d) = 100$

$a_1 - a_2 = 100$ (1)

Difference between 1000^{th} terms of these A.P.s

$(a_1 + 999d) - (a_2 + 999d) = a_1 - a_2$

From equation (1),

This difference, $a_1 - a_2 = 100$

Hence, the difference between 1000^{th} terms of these A.P. will be 100.

Question 13:

How many three digit numbers are divisible by 7

Answer:

First three-digit number that is divisible by 7 = 105

Next number = 105 + 7 = 112

Therefore, 105, 112, 119, …

All are three digit numbers which are divisible by 7 and thus, all these are terms of an A.P. having first term as 105 and common difference as 7.

The maximum possible three-digit number is 999. When we divide it by 7, the remainder will be 5. Clearly, 999 − 5 = 994 is the maximum possible three-digit number that is divisible by 7.

The series is as follows.

105, 112, 119, …, 994

Let 994 be the nth term of this A.P.

$a = 105$

$d = 7$

$a_n = 994$

$n = ?$

$a_n = a + (n - 1) d$

$994 = 105 + (n - 1) 7$

$889 = (n - 1) 7$

$(n - 1) = 127$

$n = 128$

Therefore, 128 three-digit numbers are divisible by 7.

Question 14:

How many multiples of 4 lie between 10 and 250?

Answer:

First multiple of 4 that is greater than 10 is 12. Next will be 16.

Therefore, 12, 16, 20, 24, …

All these are divisible by 4 and thus, all these are terms of an A.P. with first term as 12 and common difference as 4.

When we divide 250 by 4, the remainder will be 2. Therefore, 250 − 2 = 248 is divisible by 4.

The series is as follows.

12, 16, 20, 24, …, 248

Let 248 be the n^{th} term of this A.P.

$$a = 12$$
$$d = 4$$
$$a_n = 248$$
$$a_n = a + (n-1)d$$
$$248 = 12 + (n-1)4$$
$$\frac{236}{4} = n - 1$$
$$59 = n - 1$$
$$n = 60$$

Therefore, there are 60 multiples of 4 between 10 and 250.

Question 15:

For what value of n, are the n^{th} terms of two APs 63, 65, 67, and 3, 10, 17, … equal

Answer:

63, 65, 67, …

a = 63

$d = a_2 - a_1 = 65 - 63 = 2$

n^{th} term of this A.P. = $a_n = a + (n - 1) d$

$a_n = 63 + (n - 1) 2 = 63 + 2n - 2$

$a_n = 61 + 2n$ (1)

3, 10, 17, …

a = 3

$d = a_2 - a_1 = 10 - 3 = 7$

n^{th} term of this A.P. = 3 + (n − 1) 7

$a_n = 3 + 7n - 7$

$a_n = 7n - 4$ (2)

It is given that, n^{th} term of these A.P.s are equal to each other.

Equating both these equations, we obtain

61 + 2n = 7n − 4

61 + 4 = 5n

5n = 65

n = 13

Therefore, 13^{th} terms of both these A.P.s are equal to each other.

Question 16:

Determine the A.P. whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

Answer:

$=a_3 = 16$

$a + (3 - 1) d = 16$

$a + 2d = 16$ (1)

$a_7 - a_5 = 12$

$[a + (7 - 1) d] - [a + (5 - 1) d] = 12$

$(a + 6d) - (a + 4d) = 12$

$2d = 12$

$d = 6$

From equation (1), we obtain

$a + 2 (6) = 16$

$a + 12 = 16$

$a = 4$

Therefore, A.P. will be

4, 10, 16, 22, …

Question 17:

Find the 20th term from the last term of the A.P. 3, 8, 13, …, 253

Answer:

Given A.P. is

3, 8, 13, …, 253

Common difference for this A.P. is 5.

Therefore, this A.P. can be written in reverse order as

253, 248, 243, ..., 13, 8, 3

For this A.P.,

$a = 253$

$d = 248 - 253 = -5$

$n = 20$

$a_{20} = a + (20 - 1)\, d$

$a_{20} = 253 + (19)\,(-5)$

$a_{20} = 253 - 95$

$a = 158$

Therefore, 20th term from the last term is 158.

Question 18:

The sum of 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P.

Answer:

We know that,

$a_n = a + (n - 1)\, d$

$a_4 = a + (4 - 1)\, d$

$a_4 = a + 3d$

Similarly,

$a_8 = a + 7d$

$a_6 = a + 5d$

$a_{10} = a + 9d$

Given that, $a_4 + a_8 = 24$

$a + 3d + a + 7d = 24$

$2a + 10d = 24$

$a + 5d = 12$ (1)

$a_6 + a_{10} = 44$

$a + 5d + a + 9d = 44$

$2a + 14d = 44$

$a + 7d = 22$ (2)

On subtracting equation (1) from (2), we obtain

$2d = 22 - 12$

$2d = 10$

$d = 5$

From equation (1), we obtain

$a + 5d = 12$

$a + 5(5) = 12$

$a + 25 = 12$

$a = -13$

$a_2 = a + d = -13 + 5 = -8$

$a_3 = a_2 + d = -8 + 5 = -3$

Therefore, the first three terms of this A.P. are −13, −8, and −3.

Question 19:

Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Answer:

It can be observed that the incomes that Subba Rao obtained in various years are in A.P. as every year, his salary is increased by Rs 200.

Therefore, the salaries of each year after 1995 are

5000, 5200, 5400, …

Here, $a = 5000$

$d = 200$

Let after n^{th} year, his salary be Rs 7000.

Therefore, $a_n = a + (n - 1) d$

$7000 = 5000 + (n - 1) 200$

$200(n - 1) = 2000$

$(n - 1) = 10$

$n = 11$

Therefore, in 11th year, his salary will be Rs 7000.

Question 20:

Ramkali saved Rs 5 in the first week of a year and then increased her weekly saving by Rs 1.75. If in the n^{th} week, her week, her weekly savings become Rs 20.75, find n.

Answer:

Given that,

a = 5

d = 1.75

a_n = 20.75

n = ?

$a_n = a + (n - 1) d$

$$20.75 = 5 + (n-1)1.75$$
$$15.75 = (n-1)1.75$$
$$(n-1) = \frac{15.75}{1.75} = \frac{1575}{175}$$
$$= \frac{63}{7} = 9$$

n − 1 = 9

n = 10

Hence, n is 10.