Exercise 5.1

Question 1:

In which of the following situations, does the list of numbers involved make as arithmetic progression and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs 50 for each subsequent metre.

(iv)The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum.

Answer:

(i) It can be observed that

Taxi fare for 1^{st} km = 15

Taxi fare for first 2 km = 15 + 8 = 23

Taxi fare for first 3 km = 23 + 8 = 31

Taxi fare for first 4 km = 31 + 8 = 39

Clearly 15, 23, 31, 39 … forms an A.P. because every term is 8 more than the preceding term.

(ii) Let the initial volume of air in a cylinder be *V* lit. In each stroke, the vacuum pump removes $\frac{1}{4}$ of air remaining in the cylinder at a time. In other words, after every stroke, only $1-\frac{1}{4}=\frac{3}{4}$th part of air will remain.

Therefore, volumes will be $V, \frac{3}{4}V, \left(\frac{3}{4}\right)^2 V, \left(\frac{3}{4}\right)^3 V$.........

Clearly, it can be observed that the adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

(iii) Cost of digging for first metre = 150

Cost of digging for first 2 metres = 150 + 50 = 200

Cost of digging for first 3 metres = 200 + 50 = 250

Cost of digging for first 4 metres = 250 + 50 = 300

Clearly, 150, 200, 250, 300 … forms an A.P. because every term is 50 more than the preceding term.

(iv) We know that if Rs P is deposited at *r*% compound interest per annum for *n* years, our money will be $P\left(1+\frac{r}{100}\right)^n$ after *n* years.

Therefore, after every year, our money will be

$$10000\left(1+\frac{8}{100}\right), 10000\left(1+\frac{8}{100}\right)^2, 10000\left(1+\frac{8}{100}\right)^3, 10000\left(1+\frac{8}{100}\right)^4, \ldots$$

Clearly, adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

Question 2:

Write first four terms of the A.P. when the first term a and the common difference d are given as follows

(i) a = 10, d = 10

(ii) a = − 2, d = 0

(iii) a = 4, d = − 3

(iv) a = − 1 d = $\frac{1}{2}$

(v) a = − 1.25, d = − 0.25

Answer:

(i) $a = 10, d = 10$

Let the series be a_1, a_2, a_3, a_4, a_5 …

$a_1 = a = 10$

$a_2 = a_1 + d = 10 + 10 = 20$

$a_3 = a_2 + d = 20 + 10 = 30$

$a_4 = a_3 + d = 30 + 10 = 40$

$a_5 = a_4 + d = 40 + 10 = 50$

Therefore, the series will be 10, 20, 30, 40, 50 …

First four terms of this A.P. will be 10, 20, 30, and 40.

(ii) $a = -2, d = 0$

Let the series be a_1, a_2, a_3, a_4 …

$a_1 = a = -2$

$a_2 = a_1 + d = -2 + 0 = -2$

$a_3 = a_2 + d = -2 + 0 = -2$

$a_4 = a_3 + d = -2 + 0 = -2$

Therefore, the series will be −2, −2, −2, −2 …

First four terms of this A.P. will be −2, −2, −2 and −2.

(iii) $a = 4, d = -3$

Let the series be a_1, a_2, a_3, a_4 ...

$a_1 = a = 4$

$a_2 = a_1 + d = 4 - 3 = 1$

$a_3 = a_2 + d = 1 - 3 = -2$

$a_4 = a_3 + d = -2 - 3 = -5$

Therefore, the series will be 4, 1, −2 −5 ...

First four terms of this A.P. will be 4, 1, −2 and −5.

(iv) $a = -1$, $d = \frac{1}{2}$

Let the series be a_1, a_2, a_3, a_4 ...

$$a_1 = a = -1$$
$$a_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2}$$
$$a_3 = a_2 + d = -\frac{1}{2} + \frac{1}{2} = 0$$
$$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$$

Clearly, the series will be

$-1, -\frac{1}{2}, 0, \frac{1}{2}$ ....

First four terms of this A.P. will be $-1, -\frac{1}{2}, 0 \text{ and } \frac{1}{2}$.

(v) $a = -1.25$, $d = -0.25$

Let the series be a_1, a_2, a_3, a_4 ...

$a_1 = a = -1.25$

$a_2 = a_1 + d = -1.25 - 0.25 = -1.50$

$a_3 = a_2 + d = -1.50 - 0.25 = -1.75$

$a_4 = a_3 + d = -1.75 - 0.25 = -2.00$

Clearly, the series will be 1.25, −1.50, −1.75, −2.00 ……..

First four terms of this A.P. will be −1.25, −1.50, −1.75 and −2.00.

Question 3:

For the following A.P.s, write the first term and the common difference.

(i) 3, 1, − 1, − 3 …

(ii) − 5, − 1, 3, 7 …

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots$

(iv) 0.6, 1.7, 2.8, 3.9 …

Answer:

(i) 3, 1, −1, −3 …

Here, first term, a = 3

Common difference, d = Second term − First term

= 1 − 3 = −2

(ii) −5, −1, 3, 7 …

Here, first term, a = −5

Common difference, d = Second term − First term

= (−1) − (−5) = − 1 + 5 = 4

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots$

Here, first term, $a = \frac{1}{3}$

Common difference, d = Second term – First term

$= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9 …

Here, first term, a = 0.6

Common difference, d = Second term – First term

= 1.7 – 0.6

= 1.1

Question 4:

Which of the following are APs? If they form an A.P. find the common difference d and write three more terms.

(i) 2, 4, 8, 16 …

(ii) $2, \frac{5}{2}, 3, \frac{7}{2} \ldots$

(iii) – 1.2, – 3.2, – 5.2, – 7.2 …

(iv) – 10, – 6, – 2, 2 …

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2} \ldots$

(vi) 0.2, 0.22, 0.222, 0.2222 ….

(vii) 0, – 4, – 8, – 12 …

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \ldots$

(ix) 1, 3, 9, 27 …

(x) a, 2a, 3a, 4a …

(xi) a, a^2, a^3, a^4 … (xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$ …

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$ …

(xiv) 1^2, 3^2, 5^2, 7^2 …

(xv) 1^2, 5^2, 7^2, 73 …

Answer:

(i) 2, 4, 8, 16 …

It can be observed that

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

$a_4 - a_3 = 16 - 8 = 8$

i.e., $a_{k+1} - a_k$ is not the same every time. Therefore, the given numbers are not forming an A.P.

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}$ …

It can be observed that

$$a_2 - a = \frac{5}{2} - 2 = \frac{1}{2}$$
$$a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$$
$$a_4 - a_3 = \frac{7}{2} - 3 = \frac{1}{2}$$

i.e., $a_{k+1} - a_k$ is same every time.

Therefore, $d = \frac{1}{2}$ and the given numbers are in A.P.

Three more terms are

$$a_5 = \frac{7}{2} + \frac{1}{2} = 4$$
$$a_6 = 4 + \frac{1}{2} = \frac{9}{2}$$
$$a_7 = \frac{9}{2} + \frac{1}{2} = 5$$

(iii) −1.2, −3.2, −5.2, −7.2 …

It can be observed that

$a_2 - a_1 = (-3.2) - (-1.2) = -2$

$a_3 - a_2 = (-5.2) - (-3.2) = -2$

$a_4 - a_3 = (-7.2) - (-5.2) = -2$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, d = −2

The given numbers are in A.P.

Three more terms are

$a_5 = -7.2 - 2 = -9.2$

$a_6 = -9.2 - 2 = -11.2$

$a_7 = -11.2 - 2 = -13.2$

(iv) −10, −6, −2, 2 …

It can be observed that

$a_2 - a_1 = (-6) - (-10) = 4$

$a_3 - a_2 = (-2) - (-6) = 4$

$a_4 - a_3 = (2) - (-2) = 4$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, d = 4

The given numbers are in A.P.

Three more terms are

$a_5 = 2 + 4 = 6$

$a_6 = 6 + 4 = 10$

$a_7 = 10 + 4 = 14$

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

It can be observed that

$$a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$$
$$a_3 - a_2 = 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$$
$$a_4 - a_3 = 3 + 3\sqrt{2} - 3 - 2\sqrt{2} = \sqrt{2}$$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = \sqrt{2}$

The given numbers are in A.P.

Three more terms are

$$a_5 = 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$$
$$a_6 = 3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$$
$$a_7 = 3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$$

(vi) 0.2, 0.22, 0.222, 0.2222 ….

It can be observed that

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

$a_4 - a_3 = 0.2222 - 0.222 = 0.0002$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

(vii) 0, −4, −8, −12 …

It can be observed that

$a_2 - a_1 = (-4) - 0 = -4$

$a_3 - a_2 = (-8) - (-4) = -4$

$a_4 - a_3 = (-12) - (-8) = -4$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, d = −4

The given numbers are in A.P.

Three more terms are

$a_5 = -12 - 4 = -16$

$a_6 = -16 - 4 = -20$

$a_7 = -20 - 4 = -24$

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \ldots$

It can be observed that

$$a_2 - a_1 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$$
$$a_3 - a_2 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$$
$$a_4 - a_3 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, d = 0

The given numbers are in A.P.

Three more terms are

$$a_5 = -\frac{1}{2} - 0 = -\frac{1}{2}$$
$$a_6 = -\frac{1}{2} - 0 = -\frac{1}{2}$$
$$a_7 = -\frac{1}{2} - 0 = -\frac{1}{2}$$

(ix) 1, 3, 9, 27 …

It can be observed that

$a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

$a_4 - a_3 = 27 - 9 = 18$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

(x) a, 2a, 3a, 4a …

It can be observed that

$a_2 - a_1 = 2a - a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_4 - a_3 = 4a - 3a = a$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, d = a

The given numbers are in A.P.

Three more terms are

$a_5 = 4a + a = 5a$

$a_6 = 5a + a = 6a$

$a_7 = 6a + a = 7a$

(xi) $a, a^2, a^3, a^4 \ldots$

It can be observed that

$a_2 - a_1 = a^2 - a = a(a - 1)$

$a_3 - a_2 = a^3 - a^2 = a^2(a - 1)$

$a_4 - a_3 = a^4 - a^3 = a^3(a - 1)$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots$

It can be observed that

$$a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$$
$$a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$$
$$a_4 - a_3 = \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$$

i.e., $a_{k+1} - a_k$ is same every time.

Therefore, the given numbers are in A.P.

And, $d = \sqrt{2}$

Three more terms are

$$a_5 = \sqrt{32} + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$$
$$a_6 = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$$
$$a_7 = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \ldots$

It can be observed that

$$a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3 \times 2} - \sqrt{3} = \sqrt{3}\left(\sqrt{2} - 1\right)$$
$$a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6} = \sqrt{3}\left(\sqrt{3} - \sqrt{2}\right)$$
$$a_4 - a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - \sqrt{3 \times 3} = \sqrt{3}\left(2 - \sqrt{3}\right)$$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

(xiv) $1^2, 3^2, 5^2, 7^2 \ldots$

Or, 1, 9, 25, 49 .....

It can be observed that

$a_2 - a_1 = 9 - 1 = 8$

$a_3 - a_2 = 25 - 9 = 16$

$a_4 - a_3 = 49 - 25 = 24$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

(xv) $1^2, 5^2, 7^2, 73$ …

Or 1, 25, 49, 73 …

It can be observed that

$a_2 - a_1 = 25 - 1 = 24$

$a_3 - a_2 = 49 - 25 = 24$

$a_4 - a_3 = 73 - 49 = 24$

i.e., $a_{k+1} - a_k$ is same every time.

Therefore, the given numbers are in A.P.

And, d = 24

Three more terms are

$a_5 = 73 + 24 = 97$

$a_6 = 97 + 24 = 121$

$a_7 = 121 + 24 = 145$