Exercise 4.3

Question 1:

Find the nature of the roots of the following quadratic equations.

If the real roots exist, find them;

(I) $2x^2 - 3x + 5 = 0$

(II) $3x^2 - 4\sqrt{3}x + 4 = 0$

(III) $2x^2 - 6x + 3 = 0$

Answer :

We know that for a quadratic equation $ax^2 + bx + c = 0$, discriminant is $b^2 - 4ac$.

(A) If $b^2 - 4ac > 0$ Ã¢â€ ’ two distinct real roots

(B) If $b^2 - 4ac = 0$ Ã¢â€ ’ two equal real roots

(C) If $b^2 - 4ac < 0$ Ã¢â€ ’ no real roots

(I) $2x^2 - 3x + 5 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = -3, c = 5$

Discriminant $= b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40$

$= -31$

As $b^2 - 4ac < 0$,

Therefore, no real root is possible for the given equation.

(II) $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$$a = 3,\ b = -4\sqrt{3}, c = 4$$

Discriminant $= b^2 - 4ac = \left(-4\sqrt{3}\right)^2 - 4(3)(4)$

$= 48 - 48 = 0$

As $b^2 - 4ac = 0$,

Therefore, real roots exist for the given equation and they are equal to each other.

And the roots will be $\frac{-b}{2a}$ and $\frac{-b}{2a}$.

$$\frac{-b}{2a} = \frac{-\left(-4\sqrt{3}\right)}{2 \times 3} = \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$$

Therefore, the roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.

(III) $2x^2 - 6x + 3 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = -6, c = 3$

Discriminant $= b^2 - 4ac = (-6)^2 - 4(2)(3)$

$= 36 - 24 = 12$

As $b^2 - 4ac > 0$,

Therefore, distinct real roots exist for this equation as follows.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)}$$
$$= \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4}$$
$$= \frac{3 \pm \sqrt{3}}{2}$$

Therefore, the roots are $\frac{3+\sqrt{3}}{2}$ or $\frac{3-\sqrt{3}}{2}$.

Question 2:

Find the values of *k* for each of the following quadratic equations, so that they have two equal roots.

(I) $2x^2 + kx + 3 = 0$

(II) $kx(x - 2) + 6 = 0$

Answer :

We know that if an equation $ax^2 + bx + c = 0$ has two equal roots, its discriminant ($b^2 - 4ac$) will be 0.

(*i*) $2x^2 + kx + 3 = 0$

This is of the form $ax^2 + bx + c = 0$.

where, $a = 2, b = k$ and $c = 3$

Discriminant, $D = b^2 - 4ac$

$= k^2 - 4 \times 2 \times 3 = k^2 - 24$

For equal roots,

$D = 0$

$\Rightarrow k^2 - 24 = 0$

$\Rightarrow k^2 = 24$ or $k = \pm\sqrt{24}$

$\Rightarrow k = \pm\sqrt{4 \times 6} = \pm 2\sqrt{6}$

(ii) $kx(x-2)+6=0$

$\Rightarrow$ $kx^2-2kx+6=0$

This is of the form $ax^2+bx+c=0$,

where $a=k, b=-2k$ and $c=6$

Discriminant, $D=b^2-4ac$

$=(-2k)^2-4\times k\times 6=4k^2-24k$

For equal roots, $D=0$

$\Rightarrow$ $4k^2-24k=0 \Rightarrow k(4k-24)=0$

$\Rightarrow$ $k=0$ (not possible) or $4k-24=0$

$\Rightarrow$ $4k=24$

$\Rightarrow$ $k=\frac{24}{4}=6$

Question 3 :

Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2?

If so, find its length and breadth.

Answer :

Let the breadth of mango grove be *l*.

Length of mango grove will be 2*l*.

Area of mango grove = (2*l*) (*l*)

$= 2l^2$

$$2l^2=800$$

$$l^2=\frac{800}{2}=400$$

$$l^2-400=0$$

Comparing this equation with $al^2+bl+c=0$, we obtain a

$=1$ $b=0$, $c=400$

Discriminant $= b^2-4ac=(0)^2-4\times(1)\times(-400)=1600$

Here, $b^2-4ac>0$

Therefore, the equation will have real roots. And hence, the desired rectangular mango grove can be designed.

$$l=\pm 20$$

However, length cannot be negative.

Therefore, breadth of mango grove = 20 m

Length of mango grove = 2 × 20 = 40 m

Question 4 :

Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Answer :

Let the age of one friend be x years.

Age of the other friend will be $(20 - x)$ years.

4 years ago, age of 1st friend = $(x - 4)$ years

And, age of 2nd friend = $(20 - x - 4)$

= $(16 - x)$ years

Given that,

$(x - 4)(16 - x) = 48$

$16x - 64 - x^2 + 4x = 48$

$-x^2 + 20x - 112 = 0$

$x^2 - 20x + 112 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 1, b = -20, c = 112$

Discriminant = $b^2 - 4ac = (-20)^2 - 4(1)(112)$

$= 400 - 448 = -48$

As $b^2 - 4ac < 0$,

Therefore, no real root is possible for this equation and hence, this situation is not possible.

Question 5:

Is it possible to design a rectangular park of perimeter 80 and area 400 m^2? If so find its length and breadth.

Answer :

Let the length and breadth of the park be l and b.

Perimeter = $2(l + b) = 80$ $l + b = 40$

Or, $b = 40 - l$

Area = $l \times b = l(40 - l) = 40l - l^2$

$40l - l^2 = 400$

$l^2 - 40l + 400 = 0$

Comparing this equation with

$al^2 + bl + c = 0$, we obtain

$a = 1, b = -40, c = 400$

Discriminate = $b^2 - 4ac = (-40)^2 - 4(1)(400)$

= 1600 - 1600 = 0

As $b^2 - 4ac = 0$,

Therefore, this equation has equal real roots. And hence, this situation is possible.

Root of this equation,

$$l = -\frac{b}{2a}$$

$$l = -\frac{(-40)}{2(1)} = \frac{40}{2} = 20$$

Therefore, length of park, $l = 20$ m

And breadth of park, $b = 40 - l = 40 - 20 = 20$ m