Exercise 4.2

Question 1:

Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$ (ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ (iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Answer :

(i) $x^2 - 3x - 10$

$= x^2 - 5x + 2x - 10$

$= x(x-5) + 2(x-5)$

$= (x-5)(x+2)$

Roots of this equation are the values for which $(x-5)(x+2) = 0$

$\therefore$ $x - 5 = 0$ or $x + 2 = 0$

i.e., $x = 5$ or $x = -2$

(ii) $2x^2 + x - 6$

$= 2x^2 + 4x - 3x - 6$

$= 2x(x+2) - 3(x+2)$

$= (x+2)(2x-3)$

Roots of this equation are the values for which $(x+2)(2x-3) = 0$

$\therefore$ $x + 2 = 0$ or $2x - 3 = 0$

i.e., $x = -2$ or $x = \frac{3}{2}$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2}$

$= \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2}$

$= x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5)$

$= (\sqrt{2}x + 5)(x + \sqrt{2})$

Roots of this equation are the values for which $(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

∴ $\sqrt{2}x+5$ = 0 or $x+\sqrt{2}$ = 0

i.e., $x = \frac{-5}{\sqrt{2}}$ or $x = -\sqrt{2}$

(iv) $2x^2 - x + \frac{1}{8}$

$= \frac{1}{8}(16x^2 - 8x + 1)$

$= \frac{1}{8}(16x^2 - 4x - 4x + 1)$

$= \frac{1}{8}(4x(4x-1) - 1(4x-1))$

$= \frac{1}{8}(4x-1)^2$

Roots of this equation are the values for which $(4x-1)^2$ = 0

(v) Given: $100x^2 - 20x + 1 = 0$

$\Rightarrow \quad 100x^2 - 10x - 10x + 1 = 0$

$\Rightarrow \quad 10x(10x - 1) - 1(10x - 1) = 0$

$\Rightarrow \quad (10x - 1)(10x - 1) = 0$

Either $10x - 1 = 0$ or $10x - 1 = 0$

$\Rightarrow x = \frac{1}{10}$ or $x = \frac{1}{10}$

Hence, the roots are $\frac{1}{10}$ and $\frac{1}{10}$.

Question 2 :

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. Find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. Find out the number of toys produced on that day.

Answer :

(i) Let the number of John's marbles be x.

Therefore, number of Jivanti's marble = 45 - x

After losing 5 marbles,

Number of John's marbles = x - 5

Number of Jivanti's marbles = 45 - x - 5 = 40 - x

It is given that the product of their marbles is 124.

$$\therefore (x-5)(40-x) = 124$$
$$\Rightarrow x^2 - 45x + 324 = 0$$
$$\Rightarrow x^2 - 36x - 9x + 324 = 0$$
$$\Rightarrow x(x-36) - 9(x-36) = 0$$
$$\Rightarrow (x-36)(x-9) = 0$$

Either $x - 36$ = 0 or x - 9 = 0

i.e., x = 36 or x = 9

If the number of John's marbles = 36,

Then, number of Jivanti's marbles = 45 - 36 = 9

If number of John's marbles = 9, Then, number of Jivanti's marbles = 45 - 9 = 36 (ii) Let the number of toys produced be x.

∴ Cost of production of each toy = Rs (55 - x)

It is given that, total production of the toys = Rs 750

$$\therefore x(55-x) = 750$$
$$\Rightarrow x^2 - 55x + 750 = 0$$
$$\Rightarrow x^2 - 25x - 30x + 750 = 0$$
$$\Rightarrow x(x-25) - 30(x-25) = 0$$
$$\Rightarrow (x-25)(x-30) = 0$$

Either $x - 25$ = 0 or x - 30 = 0

i.e., x = 25 or x = 30

Hence, the number of toys will be either 25 or 30.

Question 3 :

Find two numbers whose sum is 27 and product is 182.

Answer :

Let the first number be x and the second number is 27 - x.

Therefore, their product = x (27 - x)

It is given that the product of these numbers is 182.

$$\text{Therefore, } x(27-x)=182$$
$$\Rightarrow x^2-27x+182=0$$
$$\Rightarrow x^2-13x-14x+182=0$$
$$\Rightarrow x(x-13)-14(x-13)=0$$
$$\Rightarrow (x-13)(x-14)=0$$

Either $x-13$ = 0 or x - 14 = 0

i.e., x = 13 or x = 14

If first number = 13, then

Other number = 27 - 13 = 14

If first number = 14, then

Other number = 27 - 14 = 13

Therefore, the numbers are 13 and 14.

Question 4 :

Find two consecutive positive integers, sum of whose squares is 365.

Answer :

Let the consecutive positive integers be x and x + 1.

$$\text{Given that } x^2+(x+1)^2=365$$
$$\Rightarrow x^2+x^2+1+2x=365$$
$$\Rightarrow 2x^2+2x-364=0$$
$$\Rightarrow x^2+x-182=0$$
$$\Rightarrow x^2+14x-13x-182=0$$
$$\Rightarrow x(x+14)-13(x+14)=0$$
$$\Rightarrow (x+14)(x-13)=0$$

Either x + 14 = 0 or x - 13 = 0, i.e., x = - 14 or x = 13

Since the integers are positive, x can only be 13. ∴

x + 1 = 13 + 1 = 14

Therefore, two consecutive positive integers will be 13 and 14.

Question 5 :

The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Answer :

Let the base of the right triangle be x cm.

Its altitude = (x - 7) cm

From pythagoras theorem,

$$\text{Base}^2 + \text{Altitude}^2 = \text{Hypotenuse}^2$$

$$\therefore x^2 + (x-7)^2 = 13^2$$

$$\Rightarrow x^2 + x^2 + 49 - 14x = 169$$

$$\Rightarrow 2x^2 - 14x - 120 = 0$$

$$\Rightarrow x^2 - 7x - 60 = 0$$

$$\Rightarrow x^2 - 12x + 5x - 60 = 0$$

$$\Rightarrow x(x-12) + 5(x-12) = 0$$

$$\Rightarrow (x-12)(x+5) = 0$$

Either x - 12 = 0 or x + 5 = 0, i.e., x = 12 or x = - 5 Since

sides are positive, x can only be 12.

Therefore, the base of the given triangle is 12 cm and the altitude of this triangle will be (12 - 7) cm = 5 cm.

Question 6 :

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

Answer :

Let the number of articles produced be x.

Therefore, cost of production of each article = Rs ($2x$ + 3) It

is given that the total production is Rs 90.

$$\therefore x(2x+3)=90$$
$$\Rightarrow 2x^2+3x-90=0$$
$$\Rightarrow 2x^2+15x-12x-90=0$$
$$\Rightarrow x(2x+15)-6(2x+15)=0$$
$$\Rightarrow (2x+15)(x-6)=0$$

Either $2x + 15 = 0$ or $x - 6 = 0$, i.e., $x = \frac{-15}{2}$ or $x = 6$

As the number of articles produced can only be a positive integer, therefore, x can only be 6.

Hence, number of articles produced = 6

Cost of each article = 2 × 6 + 3 = Rs 15