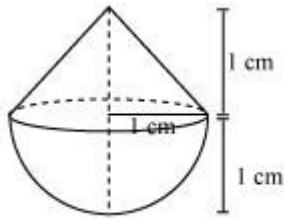**Question 1:**

A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Answer:

Given that,

Height (h) of conical part = Radius(r) of conical part = 1 cm

Radius(r) of hemispherical part = Radius of conical part (r) = 1 cm

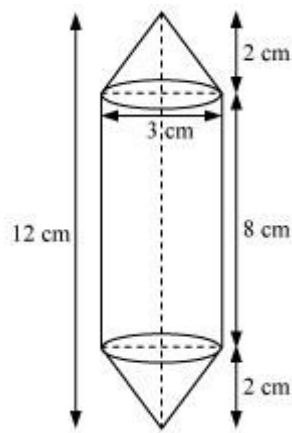Volume of solid = Volume of conical part + Volume of hemispherical part

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi(1)^2(1) + \frac{2}{3}\pi(1)^3 = \frac{2\pi}{3} + \frac{2\pi}{3} = \pi \text{ cm}^3$$

Question 2:

Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminum sheet. The diameter of the model is 3 cm and its length is 12 cm. if each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.) $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:

From the figure, it can be observed that

Height (h_1) of each conical part = 2 cm

Height (h_2) of cylindrical part = 12 − 2 × Height of conical part

= 12 − 2 ×2 = 8 cm

Radius (r) of cylindrical part = Radius of conical part = $\dfrac{3}{2}$ cm

Volume of air present in the model = Volume of cylinder + 2 × Volume of cones

$$= \pi r^2 h_2 + 2 \times \frac{1}{3}\pi r^2 h_1$$

$$= \pi \left(\frac{3}{2}\right)^2 (8) + 2 \times \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 (2)$$

$$= \pi \times \frac{9}{4} \times 8 + \frac{2}{3}\pi \times \frac{9}{4} \times 2$$
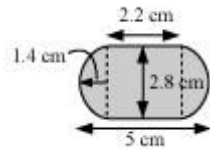
$$= 18\pi + 3\pi = 21\pi = 66 \text{ cm}^2$$

Question 3:

A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see the given figure).

$$\left[\text{Use } \pi = \frac{22}{7} \right]$$

Answer:

It can be observed that

Radius (r) of cylindrical part = Radius (r) of hemispherical part = $\dfrac{2.8}{2} = 1.4\,\text{cm}$

Length of each hemispherical part = Radius of hemispherical part = 1.4 cm

Length (h) of cylindrical part = 5 − 2 × Length of hemispherical part

= 5 − 2 × 1.4 = 2.2 cm

Volume of one gulab jamun = Vol. of cylindrical part + 2 × Vol. of hemispherical part

$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2 h + \frac{4}{3}\pi r^3$$

$$= \pi \times (1.4)^2 \times 2.2 + \frac{4}{3}\pi (1.4)^3$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times 2.2 + \frac{4}{3} \times \frac{22}{7} \times 1.4 \times 1.4 \times 1.4$$

$$= 13.552 + 11.498 = 25.05 \text{ cm}^3$$

Volume of 45 gulab jamuns = 45×25.05 = 1,127.25 cm^3

Volume of sugar syrup = 30% of volume

$$= \frac{30}{100} \times 1{,}127.25$$

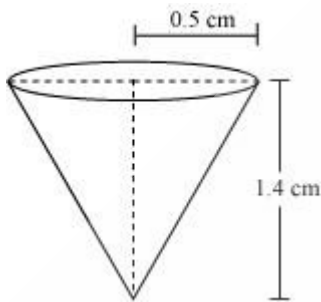$$= 338.17 \text{ cm}^3$$

$$\simeq 338 \text{ cm}^3$$

Question 4:

A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboids are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see the following figure). $\left[\text{Use } \pi = \frac{22}{7} \right]$
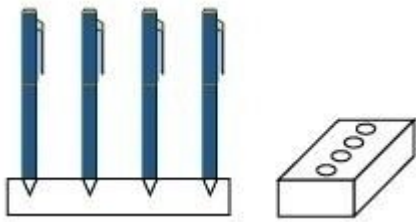
Answer:

Depth (h) of each conical depression = 1.4 cm

Radius (r) of each conical depression = 0.5 cm

Volume of wood = Volume of cuboid − 4 × Volume of cones

$$= lbh - 4 \times \frac{1}{3} \pi r^2 h$$

$$= 15 \times 10 \times 3.5 - 4 \times \frac{1}{3} \times \frac{22}{7} \times \left(\frac{1}{2} \right)^2 \times 1.4$$

$$= 525 - 1.47$$

$$= 523.53 \text{ cm}^3$$

Question 5:

A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Answer:
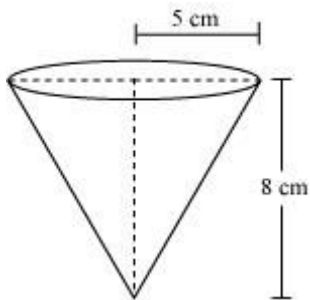
Height (h) of conical vessel = 8 cm

Radius (r_1) of conical vessel = 5 cm

Radius (r_2) of lead shots = 0.5 cm

Let n number of lead shots were dropped in the vessel.

Volume of water spilled = Volume of dropped lead shots

$$\frac{1}{4} \times \text{Volume of cone} = n \times \frac{4}{3} r_2^3$$

$$\frac{1}{4} \times \frac{1}{3} \pi r_1^2 h = n \times \frac{4}{3} \pi r_2^3$$

$$r_1^2 h = n \times 16 r_2^3$$

$$5^2 \times 8 = n \times 16 \times (0.5)^3$$

$$n = \frac{25 \times 8}{16 \times \left(\frac{1}{2}\right)^3} = 100$$

Hence, the number of lead shots dropped in the vessel is 100.

Question 6:

A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm³ of iron has approximately 8 g mass. [Use π = 3.14]

Answer:
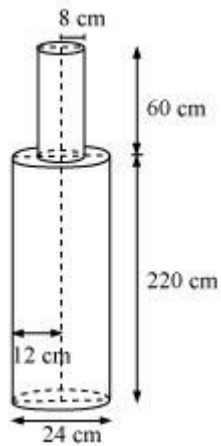
From the figure, it can be observed that

Height (h_1) of larger cylinder = 220 cm

Radius (r_1) of larger cylinder = $\dfrac{24}{2}$ = 12 cm

Height (h_2) of smaller cylinder = 60 cm

Radius (r_2) of smaller cylinder = 8 cm

$$\begin{aligned}
\text{Total volume of pole} &= \text{Volume of larger cylinder} + \text{Volume of smaller cylinder} \\
&= \pi r_1^2 h_1 + \pi r_2^2 h_2 \\
&= \pi (12)^2 \times 220 + \pi (8)^2 \times 60 \\
&= \pi [144 \times 220 + 64 \times 60] \\
&= 35520 \times 3.14 = 1,11,532.8 \text{ cm}^3
\end{aligned}$$

Mass of 1 cm³ iron = 8 g

Mass of 111532.8 cm³ iron = 111532.8 × 8 = 892262.4 g = 892.262 kg

Question 7:

A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Answer:
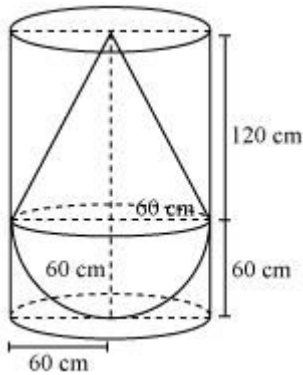
Radius (r) of hemispherical part = Radius (r) of conical part = 60 cm

Height (h_2) of conical part of solid = 120 cm

Height (h_1) of cylinder = 180 cm

Radius (r) of cylinder = 60 cm

Volume of water left = Volume of cylinder − Volume of solid

$$= \text{Volume of cylinder} - \left(\text{Volume of cone} + \text{Volume of hemisphere}\right)$$

$$= \pi r^2 h_1 - \left(\frac{1}{3}\pi r^2 h_2 + \frac{2}{3}\pi r^3\right)$$

$$= \pi(60)^2(180) - \left(\frac{1}{3}\pi(60)^2 \times 120 + \frac{2}{3}\pi(60)^3\right)$$

$$= \pi(60)^2\left[(180) - (40 + 40)\right]$$

$$= \pi(3,600)(100) = 3,60,000\pi \text{ cm}^3 = 1131428.57 \text{ cm}^3 = 1.131 \text{ m}^3$$

Question 8:

A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter o the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm³. Check whether she is correct, taking the above as the inside measurements, and π = 3.14.

Answer:
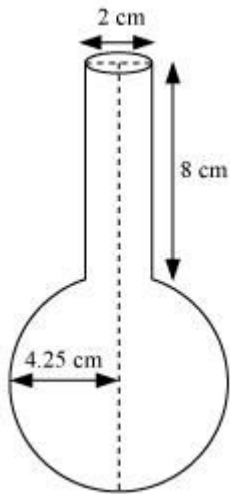
Height (h) of cylindrical part = 8 cm

Radius (r_2) of cylindrical part = $\dfrac{2}{2} = 1$ cm

Radius (r_1) spherical part = $\dfrac{8.5}{2} = 4.25$ cm

Volume of vessel = Volume of sphere + Volume of cylinder

$$= \frac{4}{3}\pi r_1^3 + \pi r_2^2 h$$

$$= \frac{4}{3}\pi \left(\frac{8.5}{2}\right)^3 + \pi(1)^2 (8)$$

$$= \frac{4}{3} \times 3.14 \times 76.765625 + 8 \times 3.14$$

$$= 321.392 + 25.12$$

$$= 346.512$$

$$= 346.51 \ cm^3$$

Hence, she is wrong.