Exercise 9.3

For each of the following differential equations in Exercise 1 to 10, find the general solution:

Q1. $\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$

A.1. The given D.E is

$$\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$$

By separable of variable,

$$\Rightarrow dy=\frac{1-\cos x}{1+\cos x}dx\left\{\begin{array}{l}\cos 2x=1-2\sin^2 x\\ =2\sin^2 x=1-\cos 2x\\ =2\sin^2\frac{x}{2}=1-\cos x\\ \cos 2x=2\cos^2 x-1\end{array}\right\}$$

$$\Rightarrow dy=\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}dx$$

$$\Rightarrow dy=\tan^2\frac{x}{2}dx$$

Integrating both sides,

$$\int dy=\int\tan^2\frac{x}{2}dx\left\{\sec^2 x=1+\tan^2\right\}$$

$$\Rightarrow y=\int\left(\sec^2\frac{x}{2}-1\right)dx$$

$$\Rightarrow y=\frac{\tan\frac{x}{2}}{\frac{1}{2}}-x+c \qquad \text{c = constant}$$

$$\Rightarrow y = 2\tan\frac{x}{2} - x + c$$ is the general solution.

Q2. $\frac{dy}{dx} = \sqrt{4-y^2}\,(-2 < y < 1)$

A2. The given D.E is

$$\frac{dy}{dx} = \sqrt{4-y^2}$$

$$\Rightarrow \frac{dy}{\sqrt{4-y^2}} = dx$$

(By separable of variables)

Integrating both sides

$$\int \frac{dy}{\sqrt{4-y^2}} = \int dx$$

$$= \sin^{-1}\frac{y}{2} = x + c$$

$$= \frac{y}{2} = \sin(x+c)$$

$$= y = 2\sin(x+c)$$ is the general solution.

Q3. $\frac{dy}{dx} + y = 1\,(y \neq 1)$

A3. Given, $\frac{dy}{dx} + y = 1$

$$\Rightarrow \frac{dy}{dx} = 1 - y = -(y-1)$$

By separable of variable,

$$\frac{dy}{(y-1)} = -dx$$

Integrating both sides,

$$\int \frac{dy}{(y-1)} = -\int dx$$

$$\Rightarrow \log|y-1| = -x + c$$

$$\Rightarrow |y-1| = e^{-x+c}$$

$$\Rightarrow y-1 = \pm e^{-x}.e^{c}$$

$\Rightarrow y = 1 + Ac$ where $A = \pm e^{c}$

Is the general solution.

Q4. $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$

A.4. Given, $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$

Dividing throughout by ‘ $\tan x \tan y$ ’ we get,

$$\frac{\sec^2 x \tan y}{\tan x \tan y} dx + \frac{\sec^2 y \tan x}{\tan x \tan y} dy = 0$$

$$\Rightarrow \frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy = 0$$

Integrating both sides we get,

$$\int \frac{\sec^2 x}{\tan x} dx + \int \frac{\sec^2 y}{\tan y} dy = \log c$$

$$\Rightarrow \log|\tan x| + \log|\tan y| = \log c \left\{ \int \frac{f^{|}(x)}{f(x)} dx - \log|f(x)| \right\}$$

$$\Rightarrow \log|(\tan x + \tan y)| = \log c$$

$$\Rightarrow \tan x \tan y = \pm c$$ is the required general solution.

Q5. $\left(e^x + e^{-x}\right)dy - \left(e^x - e^{-x}\right)dx = 0$

A.5. Given, $\left(e^x + e^{-x}\right)dy - \left(e^x - e^{-x}\right)dx = 0$

$$\Rightarrow \left(e^x + e^{-x}\right)dy = \left(e^x - e^{-x}\right)dx$$

$$\Rightarrow dy = \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$$

Integrating both sides

$$\Rightarrow \int dy = \frac{e^x - e^{-x}}{e^x + e^{-x}}dx \left\{\because \int \frac{f^{|}(x)}{f(x)}dx = \log|x|\right\}$$

$$\Rightarrow y = \log\left|e^x + e^{-x}\right| + c$$ is the required general solution.

Q6. $\frac{dy}{dx} = \left(1 + x^2\right)\left(1 + y^2\right)$

A.6. Given, $\frac{dy}{dx} = \left(1 + x^2\right)\left(1 + y^2\right)$

$$\Rightarrow \frac{dy}{\left(1 + y^2\right)} = \left(1 + x^2\right)dx$$

Integrating both sides

$$\int \frac{dy}{(1+y^2)} dy = \int (x^2+1)\, dx$$

$$\Rightarrow \tan^{-1}\frac{y}{1} = \frac{x^3}{3} + x + c$$

$$\Rightarrow \tan^{-1} y = \frac{x^3}{3} + x + c$$ is the general solution.

Q7. $y \log y dx - x dy = 0$

A.7. Given, $y \log y dx - x dy = 0$

$$\Rightarrow y \log y dx = x dy$$

$$\Rightarrow \frac{dy}{y \log y} = \frac{dx}{x}$$

Integration both sides,

$$\int \frac{dy}{y \log y} = \int \frac{dx}{x}$$

Put log $y = t \Rightarrow \frac{1}{y} = \frac{dt}{dy} \Rightarrow \frac{dy}{y} = dt$

Hence, $\int \frac{dt}{t} = \int \frac{dx}{x}$

$$\Rightarrow \log|t| = \log|x| + \log|c| = \log|xc|$$

$$\Rightarrow t = \pm xc$$

$\Rightarrow \log y = ax$ where $a = \pm c$

$\Rightarrow y = e^{ax}$ is the general solution.

Q8. $x^5 \frac{dy}{dx} = -y^5$

A.8. Given, $x^5 \frac{dy}{dx} = -y^5$

$$\Rightarrow \frac{dy}{y^5} = -\frac{dx}{x^5}$$

Integrating both sides

$$\int \frac{dy}{y^5} = -\int \frac{dx}{x^5}$$
$$\Rightarrow \int y^{-5} dy = -\int x^{-5} dx$$

$$\Rightarrow \frac{y^{-5+1}}{(-5+1)} = -\frac{x^{-5+1}}{(-5+1)} + c$$
$$\Rightarrow \frac{1}{-4y^4} = \frac{1}{4x^4} + c$$

$\Rightarrow \frac{1}{y^4} = \frac{1}{x^4} + 4c$ is the general solution.

Q9. $\frac{dy}{dx} = \sin^{-1} x$

A.9. Given, $\frac{dy}{dx} = \sin^{-1} x$

$$\Rightarrow dy = \sin^{-1} x dx$$

Integrating

$$\int dy = \int \sin^{-1} x dx$$

$$\Rightarrow y = \sin^{-1} x \int dx - \int \frac{d}{dx}\left(\sin^{-1} x\right) \int dx.dx$$

$$\Rightarrow y = x\sin^{-1} x - \int \frac{1}{\sqrt{1-x^2}} x dx + c$$

$$\Rightarrow x\sin^{-1} x - I + c$$

So, $I = \int \frac{1}{\sqrt{1-x^2}} dx = \frac{1}{2} \frac{2x}{\sqrt{1-x^2}} dx$

Put $1-x^2 = t$ So, $-2xdx = dt$

So, $I = -\frac{1}{2}\int \frac{dt}{\sqrt{t}} = -\frac{1}{2}\int t^{\frac{-1}{2}} dt = -\frac{1}{2} \frac{t^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} = -\sqrt{t}$

$$y = x\sin^{-1} x - \left(-\sqrt{1-x^2}\right) + c$$

$$y = x\sin^{-1} x + \sqrt{1-x^2} + c$$

Q.10. $e^x \tan y dx + \left(1-e^x\right)\sec^2 y dy = 0$

A.10. Given, $e^x \tan y dx + \left(1-e^x\right)\sec^2 y dy = 0$

Dividing throughout by $\left(1-e^x\right)\tan y$ we get,

$$\frac{e^x \tan y}{\left(1-e^x\right)\tan y} dx + \frac{\left(1-e^x\right)\sec^2 y}{\left(1-e^x\right)\tan y} dy = 0$$

$$= \frac{e^x}{1-e^x} dx + \frac{\sec^2 y}{\tan y} dy = 0$$

Integrating both sides

$$= \int \frac{-e^x}{1-e^x}dx + \int \frac{\sec^2 y}{\tan y}dy = c \log c$$

$$= -\log\left|1-e^x\right| + \log\left|\tan y\right| = c\log c$$

$$= \log \frac{\tan y}{1-e^x} = \log c$$

$$= \frac{\tan y}{1-e^x} = c$$

$= \tan y = \left(1-e^x\right)c$ is the general solution.

For each of the differential equations in Question 11 to 12, find a particular solution satisfying the given condition:

Q11. $\left(x^3+x^2+x+1\right)\frac{dy}{dx} = 2x^2+x$

A.11. The given D.E is $\left(x^3+x^2+x+1\right)\frac{dy}{dx} = 2x^2+x$

$$\Rightarrow dy = \left(\frac{2x^2+x}{x^3+x^2+x+1}\right)dx$$

$$\Rightarrow dy = \frac{2x^2+x}{x^2(x+1)+(x+1)}dx = \frac{2x^2+x}{(x+1)\left(x^2+1\right)}dx$$

Integrating both sides we get,

$$\int dy = \int \frac{2x^2+x}{(x+1)\left(x^2+1\right)}dx$$

Let, $\frac{2x^2+x}{(x+1)\left(x^2+1\right)} = \frac{A}{x+1} + \frac{Bx+c}{x^2+1}$

$$\Rightarrow 2x^2+2 = A\left(x^2+1\right) + (Bx+c)(x+1)$$

$$= Ax^2 + A + Bx^2 + Bx + Cx + C$$

$$= (A+B)x^2 + (B+C)x^2 + (A+C)$$

Comparing the co-efficient we get,

$$A+B=2-------(1)$$
$$B+C=1-------(2)$$
$$A+C=0-------(3)$$

Subtracting equation (1) – (2), we get

$$A+B-(B+C)=2-1$$
$$\to A-C=1$$

But from equation (3) $A=-C$ so, we get,

$$A(-C)-C=1$$
$$\to -2C=1$$
$$\to C=-\frac{1}{2}$$
$$\& A=-\left(-\frac{1}{2}\right)=\frac{1}{2}$$

And putting value of A in equation (1),

$$\frac{1}{2}+B=2$$
$$\Rightarrow B=2-\frac{1}{2}=\frac{4-1}{2}=\frac{3}{2}$$

Putting value of A,B and C in

$$\frac{2x^2+x}{(x+1)(x^2+1)}=\frac{\frac{1}{2}}{x+1}+\frac{\frac{3}{2}x-\frac{1}{2}}{x^2+1}$$
$$=\frac{1}{2(x+1)}+\frac{3}{2}\left(\frac{x}{x^2+1}\right)-\frac{1}{2}\left(\frac{1}{x^2+1}\right)$$

Hence, the integration becomes

$$\int dy=\int\frac{1}{2(x+1)}dx+\int\frac{3}{4}\left(\frac{2x}{x^2+1}\right)dx-\int\frac{1}{2}\left(\frac{1}{x^2+1}\right)dx$$
$$\Rightarrow y=\frac{1}{2}\log(x+1)+\frac{3}{4}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}\frac{x}{1}+c$$

Given, At $x=0, y=1$

Then, $1=\frac{1}{2}\log 1+\frac{3}{4}\log 1-\frac{1}{2}\tan^{-1}(0)+C$

$$\Rightarrow 1=0+0-0+C\begin{Bmatrix}\because \log 1=0\\ \tan^{-1}0-0\end{Bmatrix}$$

$$\Rightarrow c=1$$

∴ The required particular solution is:

$$y=\frac{1}{2}\log(x+1)+\frac{3}{4}\log(x^2+1)-\frac{1}{2}\tan^{-1}x+1$$

$$=\frac{1}{4}\left[2\log(x+1)+3\log(x^2+1)\right]-\frac{1}{2}\tan^{-1}x+1$$

$$=\frac{1}{4}\left[\log(x+1)^2+\log(x^2+1)^3\right]-\frac{1}{2}\tan^{-1}x+1$$

$$=\frac{1}{4}\left[\log(x+1)^2(x^2+1)^3\right]-\frac{1}{2}\tan^{-1}x+1$$

Q12. $x(x^2-1)\frac{dy}{dx}=1; y=0$ when $x=2$

A.12. The given D.E. is

$$x(x^2-1)\frac{dy}{dx}=1$$

$$\Rightarrow dy=\frac{dx}{x(x^2-1)}$$

Integrating both sides,

$$\int dy=\int\frac{dx}{x(x^2-1)}$$

$$\Rightarrow y=\int\frac{dx}{x(x^2-1)(x+1)}dx+c.$$

Let, $\frac{1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{c}{x+1}$

$$1 = A(x-1)(x+1) + B(x)(x+1) + C(x)(x-1)$$
$$= A(x^2-1) + Bx^2 + Bx + Cx^2 - Cx$$
$$= Ax^2 - A + Bx^2 + Bx + Cx^2 - Cx$$
$$= (A+B+C)x^2 + (B-C)x - A$$

Comparing the coefficient,

$$-A = 1$$
$$\Rightarrow A = -1 ------------- (1)$$
$$\Rightarrow A + B + C = 0 ---------- (2)$$
$$\Rightarrow B - C = 0$$
$$\Rightarrow B = C -------------- (3)$$

Putting equation (1) & (2) in (1) we get,

$$-1 + B + B = 0$$
$$\Rightarrow -1 + 2B = 0$$
$$\Rightarrow B = \frac{1}{2} = C$$

So, $\frac{1}{x(x-1)(x+1)} = -\frac{1}{x} + \frac{\frac{1}{2}}{x-1} + \frac{\frac{1}{2}}{x+1}$

$$= -\frac{1}{x} + \frac{1}{2(x-1)} + \frac{1}{2(x+1)}$$

Integrating becomes,

$$y = \int -\frac{1}{x}dx + \int \frac{1}{2(x-1)}dx + \int \frac{1}{2(x+1)}dx + c$$

$$= -\log(x) + \frac{1}{2}\log(x-1) + \frac{1}{2}\log(x+1) + c$$

$$= \frac{1}{2}\left[-2\log(x) + \log(x-1) + \log(x+1)\right] + c$$

$$= \frac{1}{2}\left[-\log x^2 + \log(x+1)(x-1)\right] + c$$

$$= \frac{1}{2}\log\frac{x^2-1}{x^2} + c$$

Given, $y = 0$ when $x = 2.$

Then, $0 = \frac{1}{2}\log\frac{2^2-1}{2^2} + c$

$$\Rightarrow 0 = \frac{1}{2}\log\frac{3}{4} + c$$

$$\Rightarrow c = -\frac{1}{2}\log\frac{3}{4}$$

$\therefore$ The required particular solution is

$$y = \frac{1}{2}\log\frac{x^2-1}{x^2} - \frac{1}{2}\log\frac{3}{4}$$

Q13. $\cos\left(\frac{dx}{dy}\right) = a\,(a \in R);\, y = 1$ when $x = 0$

A13. Given, D.E. is

$$\cos\frac{dy}{dx} = a$$

$$\Rightarrow \frac{dy}{dx} = \cos^{-1}(a)$$

$$\Rightarrow dy = \cos^{-1}(a)\,dx$$

Integrating both sides,

$$\int dy = \int \cos^{-1}(a)\,dx$$
$$\Rightarrow y = \cos^{-1}(a)\times x + c$$
$$\Rightarrow y = x\cos^{-1}(a)\,dx$$

Given, $y = 1, at \;\; x = 0$

Then, $1 = 0\cos^{-1}(a) + c$

$$\Rightarrow c = 1$$

$\therefore$ The required particular solution is

$$y = x\cos^{-1} a + 1$$
$$\Rightarrow y - 1 = x\cos^{-1} a$$
$$\Rightarrow \frac{y-1}{x} = \cos^{-1} a$$
$$\Rightarrow \cos\frac{y-1}{x} = a$$

Q14. $\frac{dy}{dx} = y\;\tan x; y = 1$ when $x = 0$

A14. Given, $\frac{dy}{dx} = y\tan x$

$$\Rightarrow \frac{dy}{y} = \tan x dx$$

Integrating both sides we get,

$$\int \frac{dy}{y} = \int \tan x dx$$
$$\Rightarrow \log y = \log|\sec x| + \log c$$
$$\Rightarrow \log y = \log|c \sec x|$$
$$\Rightarrow y = c_1 \sec x (where, c_1 = \pm c)$$

As, $y = 1, at, x = 0$ we have,

$$1 = c_1 \sec(0) = c \Rightarrow c = 1$$

∴ The required particular solution is $y = \sec x$.

Q15. Find the equation of the curve passing through the point (0, 0) and whose differential equation is y' = e^x sin x

A.15. The given D.E. is $y^1 = e^x \sin x$

$$dy = e^x \sin x dx$$

Integrating both sides,

$$\int dy = \int e^x \sin x dx$$
$$\Rightarrow y = I + c$$

Where, $I = \int e^x \sin x dx$

$$= \sin x \int e^x dx - \int \frac{d}{dx} \sin x \int e^x dx . dx$$
$$= \sin x . e^x - \int \cos x e^x dx$$
$$= \sin x e^x - \left\{ \cos x \int e^x dx - \int \frac{d}{dx}(\cos x) . \int I = \int e^x x dx \right\}$$
$$= \sin x . e^x - \left\{ \cos x e^x + \int \sin x e^x dx \right\}$$
$$= \sin x . e^x - \cos x e^x - I$$
$$\Rightarrow I + I = e^x (\sin x - \cos x)$$
$$\Rightarrow I = \frac{e^x}{2}(\sin x - \cos x) + c$$

Hence, $y = \frac{e^x}{2}(\sin x - \cos x) + c$

When the curve passed point (0,0),

$$y = 0, at, x = 0$$
$$\Rightarrow 0 = \frac{e^x}{2}(\sin 0 - \cos 0) + c$$
$$\Rightarrow \frac{e^0}{2}(0-1) = c$$
$$\Rightarrow c = \frac{1}{2}$$

$\therefore$ The required equation of the curve is $y = \frac{e^x}{2}(\sin x - \cos x) + \frac{1}{2}$

$$\Rightarrow 2y = e^x(\sin x - \cos x) + 1$$
$$\Rightarrow 2y - 1 = e^x(\sin x - \cos x)$$

Q.16. For the differential equation $xy\frac{dy}{dx} = (x+2)(y+2)$ find the solution curve passing through the point (1,-1)

A16.The Given D.E is

$$xy\frac{dy}{dx} = (x+2)(y+2)$$
$$\Rightarrow y\frac{dy}{y+2} = \frac{(x+2)}{2}dx$$
$$\Rightarrow \frac{y+2-2}{y+2}dy = \left(\frac{x}{x} + \frac{2}{x}\right)dx$$
$$\Rightarrow \left(1 - \frac{2}{y+2}\right)dy = \left(1 + \frac{2}{x}\right)dydx$$

Integrating both sides,

$$\int\left(1-\frac{2}{y+2}\right)dy=\int\left(1+\frac{2}{x}\right)dydx$$
$$\Rightarrow y-2\log|y+2|=x+2\log|x|+c$$
$$\Rightarrow y-\log(y+2)^2=x+\log x^2+c$$
$$\Rightarrow y-x=\log(y+2)^2+\log x^2+c$$
$$\Rightarrow y-x=\log\left[(y+2)^2.x^2\right]+c$$

A the curve passes through (-1,1) then $y=-2, at, x=1$

So, $-1-1=\log(-1+2)^2.(1)^2+c$

$$\Rightarrow -2=\log 1+c$$
$$\Rightarrow c=-2$$

$\therefore$ The required equation of curve is,

$$y-x=\log\left[(y+2)^2x^2\right]-2$$

Q.17 Find the equation of the curve passing through the point (0,-2) given that at any point (x,y) on the curve the product of the slope of its tangent and y-coordinate of the point is equal to the x-coordinate of the point.

A.17. The slope of the tangent to then curve is $\frac{dy}{dx}$

$$\frac{dy}{dx}.y=x$$
$$\Rightarrow y.dy=xdx$$

So,

Integrating both sides,

$$\int y.dy = \int xdx$$

$$\Rightarrow \int \frac{y^2}{2} = \frac{x^2}{2} + c$$

$$\Rightarrow y^2 = x^2 + A, Where, A = 2c$$

As the curve passes through (0,-2) we have,

$$(-2)^2 = 0^2 + A$$

$$\Rightarrow A = 4$$

∴ The equation of the curve is

$$y^2 = x^2 + 4$$

Q18. At any point (*x*, *y*) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (–4, –3). Find the equation of the curve given that it passes through (–2, 1).

A18. The slope of tangent is $\frac{dy}{dx}$ and slope of line joining line (-4,-3) and point say P(x,y)

$$\frac{y-(-3)}{x-(-4)} = \frac{y+3}{x+4}$$

So, $\frac{dy}{dx} = 2\left(\frac{y+3}{x+4}\right)$

$$\Rightarrow \frac{dy}{y+3} = \frac{2}{x+4}dx$$

Integrating both sides,

$$\int \frac{dy}{y+3} = \int \frac{2}{x+4} dx$$
$$\Rightarrow \log|y+3| = 2\log|x+4| + \log|c|$$
$$\Rightarrow \log|y+3| = \log(x+4)^2 + \log|c|$$
$$\Rightarrow \log|y+3| = \log\left|c(x+4)^2\right|$$
$$\Rightarrow y+3 = c_1(x+4)^2, where, c_1 = \pm c$$

Since, the curve passes through (-2,1) we get,

$$y = 1, at, x = -2$$
$$\Rightarrow 1+3 = c(-2+4)^2$$
$$\Rightarrow 4 = c \times 4$$
$$\Rightarrow c = 1$$

$\therefore$ The equation of the curve is $y+3 = (x+4)^2$

Q19. The volume of the spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after *t* seconds.

A.19. Let 'r' and U be the radius and volume of the spherical balloon.

Then, $\frac{dU}{dt} = k, \mathrm{k} = \text{constant}$

$$\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = k$$
$$\Rightarrow 4\pi r^2 \frac{dr}{dt} = k$$
$$\Rightarrow 4\pi r^2 dr = kdt$$

Integrating both sides,

$$\int 4\pi r^2 dr = \int kdt$$
$$\Rightarrow \frac{4}{3}\pi r^3 = kt + c$$

Given at t = 0, r = 3

So, $4\pi(3)^3 = c$

⇨ C = 36π

And, at t=3, r=6

So, $$\frac{4}{3}\pi(6)^3 = 3k + 36\pi\,(c = 36\pi)$$

$$\Rightarrow 288\pi - 36\pi = 3k$$
$$\Rightarrow k = \frac{252\pi}{3} = 84\pi$$

Hence, putting value of c and k in,

$$\frac{4}{3}\pi r^3 = kt + c$$, we get,

$$\frac{4}{3}\pi r^3 = 84\pi.t + 36\pi$$
$$\Rightarrow r^3 = \frac{3}{4\pi}(84\pi.t + 36\pi)$$
$$\Rightarrow r^3 = 63t + 27$$
$$\Rightarrow r = [63t + 27]^{\frac{1}{3}}$$

Q20. In a bank, principal increases continuously at the rate of *r*% per year. Find the value of *r* if Rs 100 doubles itself in 10 years ($\log_e 2 = 0.6931$).

A.20. Let P, r and t be the principal rate and time respectively.

Then, increase in principal $\frac{dP}{dt} = P \times r\%$

$$\Rightarrow \frac{dP}{dt} = P.\frac{r}{100}$$
$$\Rightarrow \frac{dP}{P} = \frac{r}{100}dt$$

Integrating both sides,

$$\int \frac{dP}{P} = \int \frac{r}{100} dt$$

$$\Rightarrow \log P = \frac{rt}{100} + c$$

$$\Rightarrow P = e^{\frac{rt}{100}+c}$$

Given at t=0,P=100

So, $$100 = e^{\frac{r\times 0}{100}+c}$$

$$\Rightarrow 100 = e^0 \times e^c$$

$$\Rightarrow e^c = 100 \left(\because e^0 = 1\right)$$

And at $t = 10, P = 2\times 100 = 200$

So, $$200 = e^{\frac{r10}{100}+c}$$

$$\Rightarrow 200 = e^{\frac{r}{10}}.e^e$$

$$\Rightarrow e^{\frac{r}{10}} = \frac{200}{100} = 2$$

$$\Rightarrow \frac{r}{10} = \log 2$$

$$\Rightarrow \frac{r}{10} = 0.6931$$

$$\Rightarrow r = 6.931$$

Hence, the rate is 6.931%

Q21. In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years

($e^{0.5} = 1.645$).

A.21. Let P and t the principal and time respectively.

Then, increase in principal $\frac{dP}{dt} = P \times 5\%$

$$\Rightarrow \frac{dP}{P} = \frac{5}{100}dt$$

Integrating both sides,

$$\Rightarrow \int \frac{dP}{P} = \int \frac{1}{20}dt$$

$$\Rightarrow \log P = \frac{t}{20} + c$$

$$\Rightarrow P = e^{\frac{t}{20}+c}$$

At, t=0,P=1000

So, $$1000 = e^{\frac{0}{20}+c}$$

$$\Rightarrow e^{c} = 1000$$

And at t=10,

$$P = e^{\frac{10}{100}+c} = e^{0.5}.e^{c}$$

$$\Rightarrow P = 1.648 \times 1000 = 1648$$

P = ₹1648

Q22. In a culture the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present.

A.22. Let ‘x’ be the number of bacteria present in instantaneous time t.

Then, $$\frac{dx}{dt} \propto x$$

$$\Rightarrow \frac{dx}{dt} = kx, where, k =$$ constant of proportionality.

$$\Rightarrow \frac{dx}{x} = kdt$$

Integrating both sides,

$$\int \frac{dx}{x} = \int kdt$$

$$\Rightarrow \log x = kt + c$$

Given, at $t = 0, x = x_0 (say) then,$

$$\log x_0 = c\left(Initial, x_0 = 100000\right)$$

So, the differential equation is

$$\log x = kt + \log x_0$$

$$\Rightarrow \log x - \log x_0 = kt$$

$$\Rightarrow \log \frac{x}{x_0} = kt$$

As the bacteria number increased by 10% in 2 hours.

The number of bacteria increased in 2hours $= 10\% \times 100000 = 10000$

Hence, at t=2,

$$x = 100000 + 10000 = 110000$$

So,

$$\log \frac{110000}{100000} = 2k$$

$$\Rightarrow k = \frac{1}{2} \log\left(\frac{11}{10}\right)$$

Hence, $log \frac{x}{x_0} = \left[\frac{1}{2} \log \frac{11}{10}\right] \times t$

$when, x = 200000,$ then we get,

$$\log\frac{200000}{100000}=\frac{1}{2}\log\frac{11}{10}\times t$$

$$\Rightarrow 2\log 2=\log\left(\frac{11}{10}\right)\times t$$

$$\Rightarrow t=\frac{2\log 2}{\log\left(\frac{11}{10}\right)}\,hours$$

Q23. The general solution of the differential equation $\frac{dy}{dx}=e^{x+y}$ **is:**

(A) $e^x + e^{-y} = C$
(B) $e^x + e^y = C$
(C) $e^{-x} + e^y = C$
(D) $e^x + e^{-y} = C$

A.23. The given D.E. is

$$\frac{dy}{dx}=e^{x+y}$$

$$\Rightarrow \frac{dy}{dx}=e^{x}.e^{y}$$

$$\Rightarrow \frac{dy}{e^{y}}=e^{x}dx$$

Integrating both sides,

$$\int\frac{dy}{e^{y}}=\int e^{x}dx$$

$$\Rightarrow \frac{e^{-y}}{-1}=e^{x}+c_1$$

$$\Rightarrow e^{-y}=-e^{x}-c_1$$

$$\Rightarrow e^{-y}+e^{x}=c, where, c=-c_1$$

$\therefore$ Option (A) is correct.