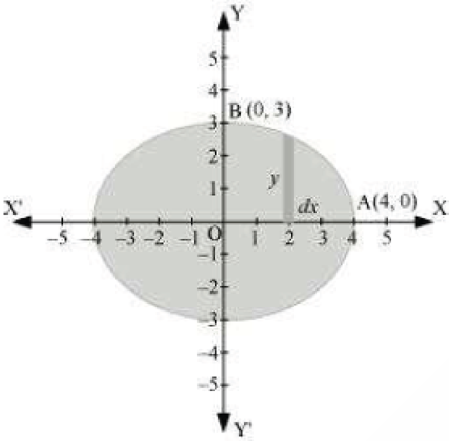
Exercise 8.1

1. Find the area of the region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

The given equation of the ellipse, $\frac{x^2}{16}+\frac{y^2}{9}=1$, can be represented as



It can be observed that the ellipse is symmetrical about *x*-axis and *y*-axis.

∴ Area bounded by ellipse = 4 × Area of OAB

$$\text{Area of OAB} = \int_0^4 y\,dx$$

$$= \int_0^4 3\sqrt{1-\frac{x^2}{16}}\,dx$$

$$= \frac{3}{4}\int_0^4 \sqrt{16-x^2}\,dx$$

$$= \frac{3}{4}\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$$

$$= \frac{3}{4}\left[2\sqrt{16-16}+8\sin^{-1}(1)-0-8\sin^{-1}(0)\right]$$

$$= \frac{3}{4}\left[\frac{8\pi}{2}\right]$$

$$= \frac{3}{4}[4\pi]$$

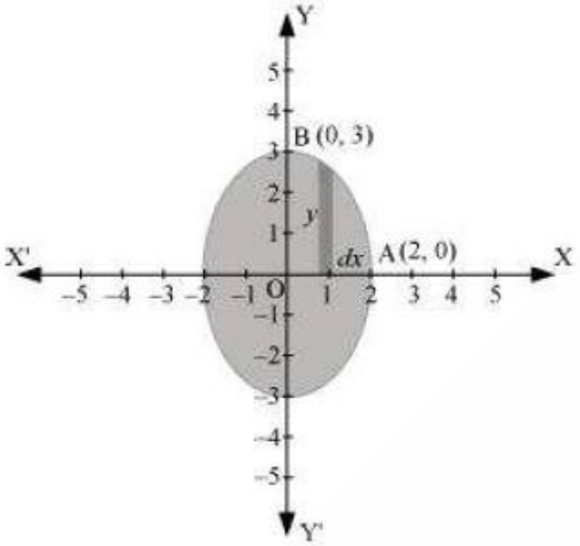
$$= 3\pi$$

Therefore, area bounded by the ellipse = 4 × 3π = 12π units

2. Find the area of the region bounded by the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$.

Answer

The given equation of the ellipse can be represented as



$$\frac{x^2}{4}+\frac{y^2}{9}=1$$

$$\Rightarrow y=3\sqrt{1-\frac{x^2}{4}} \qquad \ldots(1)$$

It can be observed that the ellipse is symmetrical about *x*-axis and *y*-axis.

∴ Area bounded by ellipse = 4 × Area OAB

$$\therefore \text{Area of OAB} = \int_0^2 y\,dx$$

$$= \int_0^2 3\sqrt{1-\frac{x^2}{4}}\,dx \qquad \text{[Using (1)]}$$

$$= \frac{3}{2}\int_0^2 \sqrt{4-x^2}\,dx$$

$$= \frac{3}{2}\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$$= \frac{3}{2}\left[\frac{2\pi}{2}\right]$$

$$= \frac{3\pi}{2}$$

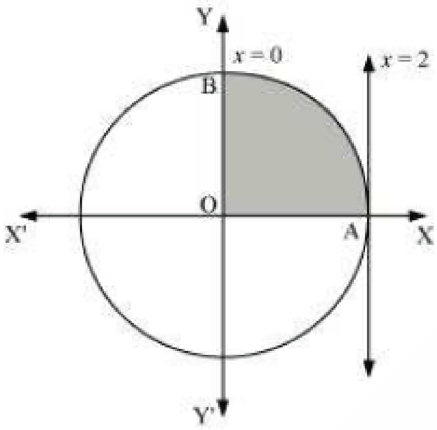
Therefore, area bounded by the ellipse = $4\times\frac{3\pi}{2}=6\pi$ units

3. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

Answer

The area bounded by the circle and the lines, $x = 0$ and $x = 2$, in the first quadrant is represented as



$$\therefore \text{Area OAB} = \int_0^2 y\,dx$$

$$= \int_0^2 \sqrt{4-x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$$= 2\left(\frac{\pi}{2}\right)$$

$$= \pi \text{ units}$$

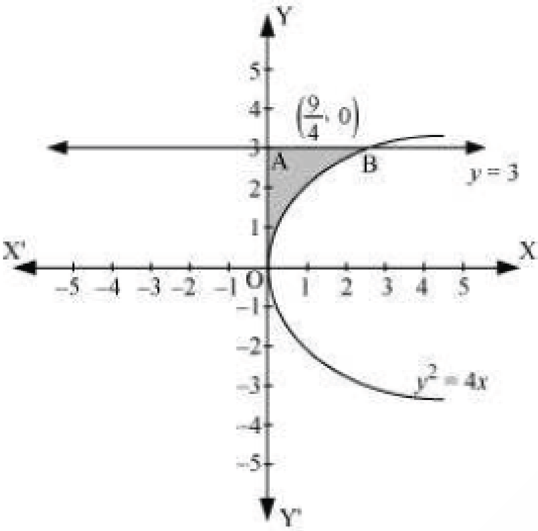
Thus, the correct answer is A.

4. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

(A) 2 (B) $\frac{9}{4}$ (C) $\frac{9}{3}$ (D) $\frac{9}{2}$

Answer

The area bounded by the curve, $y^2 = 4x$, y-axis, and $y = 3$ is represented as



$$\therefore \text{Area OAB} = \int_0^3 x\,dy$$

$$= \int_0^3 \frac{y^2}{4} dy$$

$$= \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3$$

$$= \frac{1}{12}(27)$$

$$= \frac{9}{4} \text{ units}$$

Thus, the correct answer is B.