

Exercise 7.2

Question 1:

$$\frac{2x}{1+x^2}$$

Answer

$$\text{Let } 1+x^2 = t$$

$$\therefore 2x \, dx = dt$$

$$\Rightarrow \int \frac{2x}{1+x^2} dx = \int \frac{1}{t} dt$$

$$= \log|t| + C$$

$$= \log|1+x^2| + C$$

$$= \log(1+x^2) + C$$

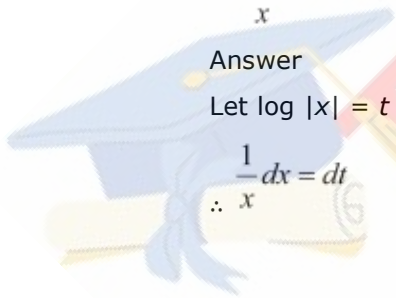
Question 2:

$$\frac{(\log x)^2}{x}$$

Answer

$$\text{Let } \log |x| = t$$

$$\therefore \frac{1}{x} dx = dt$$



$$\begin{aligned}\Rightarrow \int \frac{(\log |x|)^2}{x} dx &= \int t^2 dt \\ &= \frac{t^3}{3} + C \\ &= \frac{(\log |x|)^3}{3} + C\end{aligned}$$

Question 3:

$$\frac{1}{x + x \log x}$$

Answer

$$\frac{1}{x + x \log x} = \frac{1}{x(1 + \log x)}$$

Let $1 + \log x = t$

$$\therefore \frac{1}{x} dx = dt$$

$$\begin{aligned}\Rightarrow \int \frac{1}{x(1 + \log x)} dx &= \int \frac{1}{t} dt \\ &= \log |t| + C \\ &= \log |1 + \log x| + C\end{aligned}$$

Question 4:

$$\sin x \cdot \sin (\cos x)$$

Answer

$$\sin x \cdot \sin (\cos x)$$

$$\text{Let } \cos x = t$$

$$\therefore -\sin x \, dx = dt$$

$$\begin{aligned}\Rightarrow \int \sin x \cdot \sin (\cos x) \, dx &= - \int \sin t \, dt \\ &= -[-\cos t] + C \\ &= \cos t + C \\ &= \cos (\cos x) + C\end{aligned}$$

Question 5:

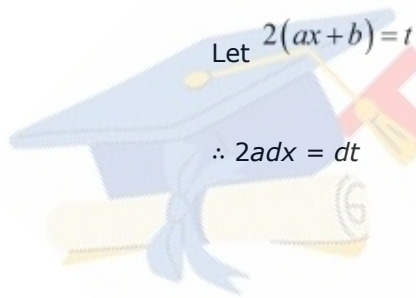
$$\sin(ax+b)\cos(ax+b)$$

Answer

$$\sin(ax+b)\cos(ax+b) = \frac{2\sin(ax+b)\cos(ax+b)}{2} = \frac{\sin 2(ax+b)}{2}$$

$$\text{Let } 2(ax+b) = t$$

$$\therefore 2adx = dt$$



$$\begin{aligned}\Rightarrow \int \frac{\sin 2(ax+b)}{2} dx &= \frac{1}{2} \int \frac{\sin t}{2a} dt \\ &= \frac{1}{4a} [-\cos t] + C \\ &= \frac{-1}{4a} \cos 2(ax+b) + C\end{aligned}$$

Question 6:

$$\sqrt{ax+b}$$

Answer

$$\text{Let } ax + b = t$$

$$\Rightarrow adx = dt$$

$$\therefore dx = \frac{1}{a} dt$$

$$\Rightarrow \int (ax+b)^{\frac{1}{2}} dx = \frac{1}{a} \int t^{\frac{1}{2}} dt$$

$$= \frac{1}{a} \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$$

$$= \frac{2}{3a} (ax+b)^{\frac{3}{2}} + C$$

Question 7:

$$x\sqrt{x+2}$$

Answer

$$\text{Let } (x+2) = t$$

$$\therefore dx = dt$$

$$\begin{aligned}\Rightarrow \int x\sqrt{x+2}dx &= \int (t-2)\sqrt{t}dt \\&= \int \left(t^{\frac{3}{2}} - 2t^{\frac{1}{2}}\right)dt \\&= \int t^{\frac{3}{2}}dt - 2 \int t^{\frac{1}{2}}dt \\&= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 2 \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C \\&= \frac{2}{5}t^{\frac{5}{2}} - \frac{4}{3}t^{\frac{3}{2}} + C \\&= \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C\end{aligned}$$

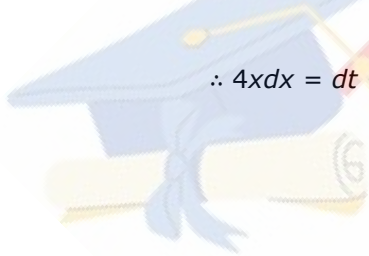
Question 8:

$$x\sqrt{1+2x^2}$$

Answer

$$\text{Let } 1 + 2x^2 = t$$

$$\therefore 4xdx = dt$$



$$\begin{aligned}\Rightarrow \int x\sqrt{1+2x^2} dx &= \int \frac{\sqrt{t} dt}{4} \\&= \frac{1}{4} \int t^{\frac{1}{2}} dt \\&= \frac{1}{4} \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C \\&= \frac{1}{6} (1+2x^2)^{\frac{3}{2}} + C\end{aligned}$$

Question 9:

$$(4x+2)\sqrt{x^2+x+1}$$

Answer

$$\text{Let } x^2 + x + 1 = t$$

$$\therefore (2x + 1)dx = dt$$

$$\begin{aligned}\int (4x+2)\sqrt{x^2+x+1} dx &= \int 2\sqrt{t} dt \\&= 2 \int \sqrt{t} dt \\&= 2 \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C \\&= \frac{4}{3} (x^2+x+1)^{\frac{3}{2}} + C\end{aligned}$$

Question 10:

$$\frac{1}{x - \sqrt{x}}$$

Answer

$$\frac{1}{x - \sqrt{x}} = \frac{1}{\sqrt{x}(\sqrt{x} - 1)}$$

Let $(\sqrt{x} - 1) = t$

$$\therefore \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow \int \frac{1}{\sqrt{x}(\sqrt{x} - 1)} dx = \int \frac{2}{t} dt$$

$$= 2 \log |t| + C$$

$$= 2 \log |\sqrt{x} - 1| + C$$

Question 11:

$$\frac{x}{\sqrt{x+4}}, x > 0$$

Answer

Let $x + 4 = t$

$$\therefore dx = dt$$

$$\begin{aligned}\int \frac{x}{\sqrt{x+4}} dx &= \int \frac{(t-4)}{\sqrt{t}} dt \\&= \int \left(\sqrt{t} - \frac{4}{\sqrt{t}} \right) dt \\&= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 4 \left(\frac{t^{\frac{1}{2}}}{\frac{1}{2}} \right) + C \\&= \frac{2}{3} (t)^{\frac{3}{2}} - 8(t)^{\frac{1}{2}} + C \\&= \frac{2}{3} t \cdot t^{\frac{1}{2}} - 8t^{\frac{1}{2}} + C \\&= \frac{2}{3} t^{\frac{1}{2}} (t-12) + C \\&= \frac{2}{3} (x+4)^{\frac{1}{2}} (x+4-12) + C \\&= \frac{2}{3} \sqrt{x+4} (x-8) + C\end{aligned}$$

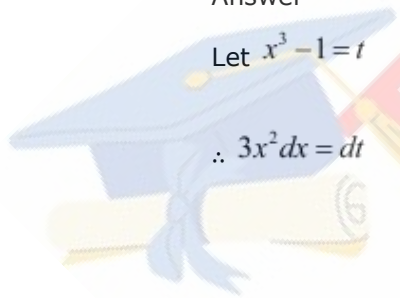
Question 12:

$$(x^3 - 1)^{\frac{1}{3}} x^5$$

Answer

Let $x^3 - 1 = t$

$\therefore 3x^2 dx = dt$



$$\begin{aligned}\Rightarrow \int (x^3 - 1)^{\frac{1}{3}} x^5 dx &= \int (x^3 - 1)^{\frac{1}{3}} x^3 \cdot x^2 dx \\&= \int t^{\frac{1}{3}} (t+1) \frac{dt}{3} \\&= \frac{1}{3} \int \left(t^{\frac{4}{3}} + t^{\frac{1}{3}} \right) dt \\&= \frac{1}{3} \left[\frac{t^{\frac{7}{3}}}{\frac{7}{3}} + \frac{t^{\frac{4}{3}}}{\frac{4}{3}} \right] + C \\&= \frac{1}{3} \left[\frac{3}{7} t^{\frac{7}{3}} + \frac{3}{4} t^{\frac{4}{3}} \right] + C \\&= \frac{1}{7} (x^3 - 1)^{\frac{7}{3}} + \frac{1}{4} (x^3 - 1)^{\frac{4}{3}} + C\end{aligned}$$

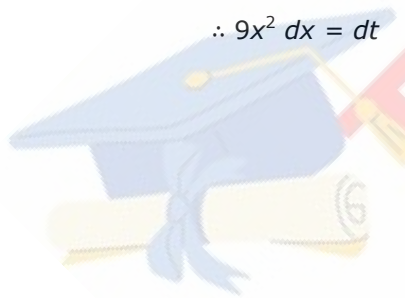
Question 13:

$$\frac{x^2}{(2+3x^3)^3}$$

Answer

Let $2+3x^3 = t$

$$\therefore 9x^2 dx = dt$$



$$\begin{aligned}
 \Rightarrow \int \frac{x^2}{(2+3x^3)^3} dx &= \frac{1}{9} \int \frac{dt}{(t)^3} \\
 &= \frac{1}{9} \left[\frac{t^{-2}}{-2} \right] + C \\
 &= \frac{-1}{18} \left(\frac{1}{t^2} \right) + C \\
 &= \frac{-1}{18(2+3x^3)^2} + C
 \end{aligned}$$

Question 14:

$$\frac{1}{x(\log x)^m}, x > 0$$

Answer

Let $\log x = t$

$$\therefore \frac{1}{x} dx = dt$$

$$\begin{aligned}
 \Rightarrow \int \frac{1}{x(\log x)^m} dx &= \int \frac{dt}{(t)^m} \\
 &= \left(\frac{t^{-m+1}}{1-m} \right) + C \\
 &= \frac{(\log x)^{1-m}}{(1-m)} + C
 \end{aligned}$$

Question 15:

$$\frac{x}{9-4x^2}$$

Answer

Let $9 - 4x^2 = t$

$$\therefore -8x \, dx = dt$$

$$\begin{aligned}\Rightarrow \int \frac{x}{9-4x^2} dx &= \frac{-1}{8} \int \frac{1}{t} dt \\ &= \frac{-1}{8} \log|t| + C \\ &= \frac{-1}{8} \log|9-4x^2| + C\end{aligned}$$

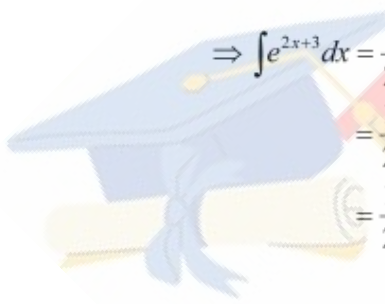
Question 16:

$$e^{2x+3}$$

Answer

Let $2x+3 = t$

$$\therefore 2dx = dt$$


$$\begin{aligned}\Rightarrow \int e^{2x+3} dx &= \frac{1}{2} \int e^t dt \\ &= \frac{1}{2} (e^t) + C \\ &= \frac{1}{2} e^{(2x+3)} + C\end{aligned}$$

Question 17:

$$\frac{x}{e^{x^2}}$$

Answer

Let $x^2 = t$

$$\therefore 2x dx = dt$$

$$\Rightarrow \int \frac{x}{e^{x^2}} dx = \frac{1}{2} \int \frac{1}{e^t} dt$$

$$= \frac{1}{2} \int e^{-t} dt$$

$$= \frac{1}{2} \left(\frac{e^{-t}}{-1} \right) + C$$

$$= -\frac{1}{2} e^{-x^2} + C$$

$$= \frac{-1}{2e^{x^2}} + C$$

Question 18:

$$\frac{e^{\tan^{-1} x}}{1+x^2}$$

Answer

Let $\tan^{-1} x = t$

$$\therefore \frac{1}{1+x^2} dx = dt$$

$$\begin{aligned}\Rightarrow \int \frac{e^{\tan^{-1}x}}{1+x^2} dx &= \int e^t dt \\ &= e^t + C \\ &= e^{\tan^{-1}x} + C\end{aligned}$$

Question 19:

$$\frac{e^{2x}-1}{e^{2x}+1}$$

Answer

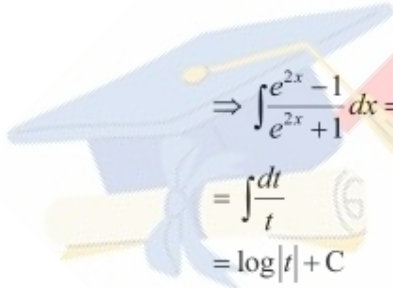
$$\frac{e^{2x}-1}{e^{2x}+1}$$

Dividing numerator and denominator by e^x , we obtain

$$\frac{\frac{(e^{2x}-1)}{e^x}}{\frac{(e^{2x}+1)}{e^x}} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Let $e^x + e^{-x} = t$

$$\therefore (e^x - e^{-x}) dx = dt$$


$$\begin{aligned}\Rightarrow \int \frac{e^{2x}-1}{e^{2x}+1} dx &= \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx \\ &= \int \frac{dt}{t} \\ &= \log|t| + C \\ &= \log|e^x + e^{-x}| + C\end{aligned}$$

Question 20:

$$\frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$$

Answer

Let $e^{2x} + e^{-2x} = t$

$$\therefore (2e^{2x} - 2e^{-2x})dx = dt$$

$$\Rightarrow 2(e^{2x} - e^{-2x})dx = dt$$

$$\Rightarrow \int \left(\frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} \right) dx = \int \frac{dt}{2t}$$

$$= \frac{1}{2} \int \frac{1}{t} dt$$

$$= \frac{1}{2} \log|t| + C$$

$$= \frac{1}{2} \log|e^{2x} + e^{-2x}| + C$$

Question 21:

$$\tan^2(2x-3)$$

Answer

$$\tan^2(2x-3) = \sec^2(2x-3) - 1$$

Let $2x - 3 = t$

$$\therefore 2dx = dt$$

$$\begin{aligned}\Rightarrow \int \tan^2(2x-3) dx &= \int [\sec^2(2x-3) - 1] dx \\&= \frac{1}{2} \int (\sec^2 t) dt - \int 1 dx \\&= \frac{1}{2} \int \sec^2 t dt - \int 1 dx \\&= \frac{1}{2} \tan t - x + C \\&= \frac{1}{2} \tan(2x-3) - x + C\end{aligned}$$

Question 22:

$$\sec^2(7-4x)$$

Answer

$$\text{Let } 7 - 4x = t$$

$$\therefore -4dx = dt$$

$$\begin{aligned}\therefore \int \sec^2(7-4x) dx &= \frac{-1}{4} \int \sec^2 t dt \\&= \frac{-1}{4} (\tan t) + C \\&= \frac{-1}{4} \tan(7-4x) + C\end{aligned}$$

Question 23:

$$\frac{\sin^{-1} x}{\sqrt{1-x^2}}$$

Answer

$$\text{Let } \sin^{-1} x = t$$

$$\therefore \frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\Rightarrow \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \int t dt$$

$$= \frac{t^2}{2} + C$$

$$= \frac{(\sin^{-1} x)^2}{2} + C$$

Question 24:

$$\frac{2 \cos x - 3 \sin x}{6 \cos x + 4 \sin x}$$

Answer

$$\frac{2 \cos x - 3 \sin x}{6 \cos x + 4 \sin x} = \frac{2 \cos x - 3 \sin x}{2(3 \cos x + 2 \sin x)}$$

$$\text{Let } 3 \cos x + 2 \sin x = t$$

$$\therefore (-3 \sin x + 2 \cos x) dx = dt$$

$$\int \frac{2 \cos x - 3 \sin x}{6 \cos x + 4 \sin x} dx = \int \frac{dt}{2t}$$

$$= \frac{1}{2} \int \frac{1}{t} dt$$

$$= \frac{1}{2} \log |t| + C$$

$$= \frac{1}{2} \log |2 \sin x + 3 \cos x| + C$$

Question 25:

$$\frac{1}{\cos^2 x (1 - \tan x)^2}$$

Answer

$$\frac{1}{\cos^2 x (1 - \tan x)^2} = \frac{\sec^2 x}{(1 - \tan x)^2}$$

Let $(1 - \tan x) = t$

$$\therefore -\sec^2 x dx = dt$$

$$\begin{aligned}\Rightarrow \int \frac{\sec^2 x}{(1 - \tan x)^2} dx &= \int \frac{-dt}{t^2} \\ &= -\int t^{-2} dt \\ &= +\frac{1}{t} + C \\ &= \frac{1}{(1 - \tan x)} + C\end{aligned}$$

Question 26:

$$\frac{\cos \sqrt{x}}{\sqrt{x}}$$

Answer

Let $\sqrt{x} = t$

$$\therefore \frac{1}{2\sqrt{x}} dx = dt$$

$$\begin{aligned}\Rightarrow \int \frac{\cos \sqrt{x}}{\sqrt{x}} dx &= 2 \int \cos t \, dt \\ &= 2 \sin t + C \\ &= 2 \sin \sqrt{x} + C\end{aligned}$$

Question 27:

$$\sqrt{\sin 2x} \cos 2x$$

Answer

$$\text{Let } \sin 2x = t$$

$$\therefore 2 \cos 2x \, dx = dt$$

$$\begin{aligned}\Rightarrow \int \sqrt{\sin 2x} \cos 2x \, dx &= \frac{1}{2} \int \sqrt{t} \, dt \\ &= \frac{1}{2} \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C \\ &= \frac{1}{3} t^{\frac{3}{2}} + C \\ &= \frac{1}{3} (\sin 2x)^{\frac{3}{2}} + C\end{aligned}$$

Question 28:

$$\frac{\cos x}{\sqrt{1 + \sin x}}$$

Answer

$$\text{Let } 1 + \sin x = t$$

$$\therefore \cos x \, dx = dt$$

$$\begin{aligned}\Rightarrow \int \frac{\cos x}{\sqrt{1+\sin x}} \, dx &= \int \frac{dt}{\sqrt{t}} \\ &= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= 2\sqrt{t} + C \\ &= 2\sqrt{1+\sin x} + C\end{aligned}$$

Question 29:

$$\cot x \log \sin x$$

Answer

$$\text{Let } \log \sin x = t$$

$$\Rightarrow \frac{1}{\sin x} \cdot \cos x \, dx = dt$$

$$\therefore \cot x \, dx = dt$$

$$\begin{aligned}\Rightarrow \int \cot x \log \sin x \, dx &= \int t \, dt \\ &= \frac{t^2}{2} + C \\ &= \frac{1}{2} (\log \sin x)^2 + C\end{aligned}$$

Question 30:

$$\frac{\sin x}{1+\cos x}$$

Answer

$$\text{Let } 1 + \cos x = t$$

$$\therefore -\sin x \, dx = dt$$

$$\begin{aligned}\Rightarrow \int \frac{\sin x}{1 + \cos x} \, dx &= \int -\frac{dt}{t} \\ &= -\log|t| + C \\ &= -\log|1 + \cos x| + C\end{aligned}$$

Question 31:

$$\frac{\sin x}{(1 + \cos x)^2}$$

Answer

$$\text{Let } 1 + \cos x = t$$

$$\therefore -\sin x \, dx = dt$$

$$\begin{aligned}\Rightarrow \int \frac{\sin x}{(1 + \cos x)^2} \, dx &= \int -\frac{dt}{t^2} \\ &= -\int t^{-2} \, dt \\ &= \frac{1}{t} + C \\ &= \frac{1}{1 + \cos x} + C\end{aligned}$$

Question 32:

$$\frac{1}{1 + \cot x}$$

Answer

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{1 + \cot x} dx \\
 &= \int \frac{1}{1 + \frac{\cos x}{\sin x}} dx \\
 &= \int \frac{\sin x}{\sin x + \cos x} dx \\
 &= \frac{1}{2} \int \frac{2 \sin x}{\sin x + \cos x} dx \\
 &= \frac{1}{2} \int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x + \cos x)} dx \\
 &= \frac{1}{2} \int 1 dx + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx \\
 &= \frac{1}{2} (x) + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx
 \end{aligned}$$

$$\text{Let } \sin x + \cos x = t \Rightarrow (\cos x - \sin x) dx = dt$$

$$\begin{aligned}
 \therefore I &= \frac{x}{2} + \frac{1}{2} \int \frac{-(dt)}{t} \\
 &= \frac{x}{2} - \frac{1}{2} \log |t| + C \\
 &= \frac{x}{2} - \frac{1}{2} \log |\sin x + \cos x| + C
 \end{aligned}$$

Question 33:

$$\frac{1}{1 - \tan x}$$

Answer

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{1 - \tan x} dx \\
 &= \int \frac{1}{1 - \frac{\sin x}{\cos x}} dx \\
 &= \int \frac{\cos x}{\cos x - \sin x} dx \\
 &= \frac{1}{2} \int \frac{2 \cos x}{\cos x - \sin x} dx \\
 &= \frac{1}{2} \int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{(\cos x - \sin x)} dx \\
 &= \frac{1}{2} \int 1 dx + \frac{1}{2} \int \frac{\cos x + \sin x}{\cos x - \sin x} dx \\
 &= \frac{x}{2} + \frac{1}{2} \int \frac{\cos x + \sin x}{\cos x - \sin x} dx
 \end{aligned}$$

Put $\cos x - \sin x = t \Rightarrow (-\sin x - \cos x) dx = dt$

$$\begin{aligned}
 \therefore I &= \frac{x}{2} + \frac{1}{2} \int \frac{-(dt)}{t} \\
 &= \frac{x}{2} - \frac{1}{2} \log |t| + C \\
 &= \frac{x}{2} - \frac{1}{2} \log |\cos x - \sin x| + C
 \end{aligned}$$

Question 34:

$$\frac{\sqrt{\tan x}}{\sin x \cos x}$$

Answer

$$\begin{aligned}\text{Let } I &= \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx \\&= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx \\&= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx \\&= \int \frac{\sec^2 x dx}{\sqrt{\tan x}}\end{aligned}$$

$$\text{Let } \tan x = t \Rightarrow \sec^2 x dx = dt$$

$$\begin{aligned}\therefore I &= \int \frac{dt}{\sqrt{t}} \\&= 2\sqrt{t} + C \\&= 2\sqrt{\tan x} + C\end{aligned}$$

Question 35:

$$\frac{(1 + \log x)^2}{x}$$

Answer

$$\text{Let } 1 + \log x = t$$

$$\therefore \frac{1}{x} dx = dt$$

$$\begin{aligned}\Rightarrow \int \frac{(1 + \log x)^2}{x} dx &= \int t^2 dt \\&= \frac{t^3}{3} + C \\&= \frac{(1 + \log x)^3}{3} + C\end{aligned}$$

Question 36:

$$\frac{(x+1)(x+\log x)^2}{x}$$

Answer

$$\frac{(x+1)(x+\log x)^2}{x} = \left(\frac{x+1}{x}\right)(x+\log x)^2 = \left(1+\frac{1}{x}\right)(x+\log x)^2$$

Let $(x+\log x) = t$

$$\therefore \left(1+\frac{1}{x}\right)dx = dt$$

$$\begin{aligned}\Rightarrow \int \left(1+\frac{1}{x}\right)(x+\log x)^2 dx &= \int t^2 dt \\ &= \frac{t^3}{3} + C \\ &= \frac{1}{3}(x+\log x)^3 + C\end{aligned}$$

Question 37:

$$\frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}$$

Answer

Let $x^4 = t$

$$\therefore 4x^3 dx = dt$$

$$\Rightarrow \int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8} dx = \frac{1}{4} \int \frac{\sin(\tan^{-1} t)}{1+t^2} dt \quad \dots(1)$$

Let $\tan^{-1} t = u$

$$\therefore \frac{1}{1+t^2} dt = du$$

From (1), we obtain

$$\begin{aligned} \int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8} dx &= \frac{1}{4} \int \sin u \, du \\ &= \frac{1}{4} (-\cos u) + C \end{aligned}$$

$$= \frac{-1}{4} \cos(\tan^{-1} t) + C$$

$$= \frac{-1}{4} \cos(\tan^{-1} x^4) + C$$

Question 38:

$$\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx \quad \text{equals}$$

- (A) $10^x - x^{10} + C$ (B) $10^x + x^{10} + C$
(C) $(10^x - x^{10})^{-1} + C$ (D) $\log(10^x + x^{10}) + C$

Answer

Let $x^{10} + 10^x = t$

$$\therefore (10x^9 + 10^x \log_e 10) dx = dt$$

$$\begin{aligned}\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx &= \int \frac{dt}{t} \\ &= \log t + C \\ &= \log(10^x + x^{10}) + C\end{aligned}$$

Hence, the correct Answer is D.

Question 39:

$$\int \frac{dx}{\sin^2 x \cos^2 x} \text{ equals}$$

- A.** $\tan x + \cot x + C$
- B.** $\tan x - \cot x + C$
- C.** $\tan x \cot x + C$
- D.** $\tan x - \cot 2x + C$

Answer

$$\begin{aligned}\text{Let } I &= \int \frac{dx}{\sin^2 x \cos^2 x} \\ &= \int \frac{1}{\sin^2 x \cos^2 x} dx \\ &= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx \\ &= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx \\ &= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx \\ &= \tan x - \cot x + C\end{aligned}$$

Hence, the correct Answer is B.