Exercise 7.10

Question 1:

$$\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

Answer

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x \, dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2} - x\right) dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^2 x\right) dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Question 2:

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Answer

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$\text{Let } I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \qquad \ldots(1)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos}}{\sqrt{\cos}+\sqrt{\sin x}}dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}} 1\cdot dx$$

$$\Rightarrow 2I=[x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

Question 3:

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x}$$

Answer

Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \quad ...(1)$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)} dx \quad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \quad ...(2)$

Adding (1) and (2), we obtain

$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$

$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$

$\Rightarrow 2I = \frac{\pi}{2}$

$\Rightarrow I = \frac{\pi}{4}$

Question 4:

$\int_0^{\frac{\pi}{2}} \frac{\cos^5 x dx}{\sin^5 x + \cos^5 x}$

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^5\left(\frac{\pi}{2} - x\right)}{\sin^5\left(\frac{\pi}{2} - x\right) + \cos^5\left(\frac{\pi}{2} - x\right)} dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Question 5:

$$\int_{-5}^{5} |x+2|\,dx$$

Answer

$$\text{Let } I = \int_{-5}^{5} |x+2|\,dx$$

It can be seen that $(x + 2) \leq 0$ on $[-5, -2]$ and $(x + 2) \geq 0$ on $[-2, 5]$.

$$\therefore I=\int_{-5}^{-2}-(x+2)dx+\int_{-2}^{5}(x+2)dx \qquad \left(\int_a^b f(x)=\int_a^c f(x)+\int_c^b f(x)\right)$$

$$I=-\left[\frac{x^2}{2}+2x\right]_{-5}^{-2}+\left[\frac{x^2}{2}+2x\right]_{-2}^{5}$$

$$=-\left[\frac{(-2)^2}{2}+2(-2)-\frac{(-5)^2}{2}-2(-5)\right]+\left[\frac{(5)^2}{2}+2(5)-\frac{(-2)^2}{2}-2(-2)\right]$$

$$=-\left[2-4-\frac{25}{2}+10\right]+\left[\frac{25}{2}+10-2+4\right]$$

$$=-2+4+\frac{25}{2}-10+\frac{25}{2}+10-2+4$$

$$=29$$

Question 6:

$$\int_2^8 |x-5|\,dx$$

Answer

$$\text{Let } I=\int_2^8 |x-5|\,dx$$

It can be seen that $(x-5)\le 0$ on [2, 5] and $(x-5)\ge 0$ on [5, 8].

$$I=\int_2^5 -(x-5)dx+\int_5^8 (x-5)dx \qquad \left(\int_a^b f(x)=\int_a^c f(x)+\int_c^b f(x)\right)$$

$$=-\left[\frac{x^2}{2}-5x\right]_2^5+\left[\frac{x^2}{2}-5x\right]_5^8$$

$$=-\left[\frac{25}{2}-25-2+10\right]+\left[32-40-\frac{25}{2}+25\right]$$

$$=9$$

Question 7:

$$\int_0^1 x(1-x)^n\,dx$$

Answer

$$\text{Let } I = \int_0^1 x(1-x)^n\,dx$$

$$\therefore I = \int_0^1 (1-x)\left(1-(1-x)\right)^n dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$= \int_0^1 (1-x)(x)^n\,dx$$

$$= \int_0^1 \left(x^n - x^{n+1}\right)dx$$

$$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$$

$$= \left[\frac{1}{n+1} - \frac{1}{n+2}\right]$$

$$= \frac{(n+2)-(n+1)}{(n+1)(n+2)}$$

$$= \frac{1}{(n+1)(n+2)}$$

Question 8:

$$\int_0^{\frac{\pi}{4}} \log\,(1+\tan x)dx$$

Answer

Let $I=\int_0^{\frac{\pi}{4}}\log\left(1+\tan x\right)dx$ ...(1)

$\therefore I=\int_0^{\frac{\pi}{4}}\log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$ $\left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log\left\{1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right\}dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log\left\{1+\frac{1-\tan x}{1+\tan}\right\}dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log\frac{2}{(1+\tan x)}dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log 2\,dx-\int_0^{\frac{\pi}{4}}\log\left(1+\tan x\right)dx$$

$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log 2dx-I$ [From (1)]

$$\Rightarrow 2I=\left[x\log 2\right]_0^{\frac{\pi}{4}}$$

$$\Rightarrow 2I=\frac{\pi}{4}\log 2$$

$$\Rightarrow I=\frac{\pi}{8}\log 2$$

Question 9:

$$\int_0^2 x\sqrt{2-x}\,dx$$

Answer

Let $I = \int_0^2 x\sqrt{2-x}dx$

$$I = \int_0^2 (2-x)\sqrt{x}dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$= \int_0^2 \left\{2x^{\frac{1}{2}} - x^{\frac{3}{2}}\right\}dx$$

$$= \left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$$

$$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^2$$

$$= \frac{4}{3}(2)^{\frac{3}{2}} - \frac{2}{5}(2)^{\frac{5}{2}}$$

$$= \frac{4\times 2\sqrt{2}}{3} - \frac{2}{5}\times 4\sqrt{2}$$

$$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$$

$$= \frac{40\sqrt{2} - 24\sqrt{2}}{15}$$

$$= \frac{16\sqrt{2}}{15}$$

Question 10:

$$\int_0^{\frac{\pi}{2}} \left(2\log\sin x - \log\sin 2x\right)dx$$

Answer

Let $I = \int_0^{\frac{\pi}{2}} (2\log \sin x - \log \sin 2x)\, dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \{2\log \sin x - \log(2\sin x \cos x)\}\, dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \{2\log \sin x - \log \sin x - \log \cos x - \log 2\}\, dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \{\log \sin x - \log \cos x - \log 2\}\, dx$ ...(1)

It is known that, $\left(\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx\right)$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \{\log \cos x - \log \sin x - \log 2\}\, dx$ ...(2)

Adding (1) and (2), we obtain

$2I = \int_0^{\frac{\pi}{2}} (-\log 2 - \log 2)\, dx$

$\Rightarrow 2I = -2\log 2 \int_0^{\frac{\pi}{2}} 1\, dx$

$\Rightarrow I = -\log 2\left[\frac{\pi}{2}\right]$

$\Rightarrow I = \frac{\pi}{2}(-\log 2)$

$\Rightarrow I = \frac{\pi}{2}\left[\log \frac{1}{2}\right]$

$\Rightarrow I = \frac{\pi}{2} \log \frac{1}{2}$

Question 11:

$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\, dx$

Answer

Let $I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\, dx$

As $\sin^2(-x) = (\sin(-x))^2 = (-\sin x)^2 = \sin^2 x$, therefore, $\sin^2 x$ is an even function.

It is known that if $f(x)$ is an even function, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

$$I = 2\int_0^{\frac{\pi}{2}} \sin^2 x\,dx$$

$$= 2\int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}\,dx$$

$$= \int_0^{\frac{\pi}{2}} (1-\cos 2x)\,dx$$

$$= \left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

Question 12:

$$\int_0^{\pi} \frac{x\,dx}{1+\sin x}$$

Answer

$$\text{Let } I = \int_0^{\pi} \frac{x\,dx}{1+\sin x} \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin(\pi - x)}\,dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin x}\,dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\pi}\frac{\pi}{1+\sin x}dx$$

$$\Rightarrow 2I=\pi\int_0^{\pi}\frac{(1-\sin x)}{(1+\sin x)(1-\sin x)}dx$$

$$\Rightarrow 2I=\pi\int_0^{\pi}\frac{1-\sin x}{\cos^2 x}dx$$

$$\Rightarrow 2I=\pi\int_0^{\pi}\left\{\sec^2 x-\tan x\sec x\right\}dx$$

$$\Rightarrow 2I=\pi\left[\tan x-\sec x\right]_0^{\pi}$$

$$\Rightarrow 2I=\pi[2]$$

$$\Rightarrow I=\pi$$

Question 13:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^7 x\,dx$$

Answer

$$\text{Let } I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^7 x dx \qquad \ldots(1)$$

As $\sin^7(-x) = (\sin(-x))^7 = (-\sin x)^7 = -\sin^7 x$, therefore, $\sin^2 x$ is an odd function.

It is known that, if $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)dx=0$

$$\therefore I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^7 x\,dx=0$$

Question 14:

$$\int_0^{2\pi}\cos^5 xdx$$

Answer

$$\text{Let } I=\int_0^{2\pi}\cos^5 xdx \qquad \ldots(1)$$

$$\cos^5(2\pi-x)=\cos^5 x$$

It is known that,

$$\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx, \text{ if } f(2a-x) = f(x)$$
$$= 0 \text{ if } f(2a-x) = -f(x)$$

$$\therefore I = 2\int_0^{\pi} \cos^5 x dx$$

$$\Rightarrow I = 2(0) = 0 \qquad \left[\cos^5(\pi - x) = -\cos^5 x\right]$$

Question 15:

$$\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$$

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx \qquad ...(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{0}{1 + \sin x \cos x} dx$$

$$\Rightarrow I = 0$$

Question 16:

$$\int_0^{\pi} \log(1 + \cos x) dx$$

Answer

$$\text{Let } I = \int_0^{\pi} \log(1 + \cos x) dx \qquad ...(1)$$

$$\Rightarrow I = \int_0^{\pi} \log(1 + \cos(\pi - x)) dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\pi} \log(1 - \cos x) dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi} \{\log(1+\cos x) + \log(1-\cos x)\} dx$$

$$\Rightarrow 2I = \int_0^{\pi} \log(1-\cos^2 x) dx$$

$$\Rightarrow 2I = \int_0^{\pi} \log \sin^2 x \, dx$$

$$\Rightarrow 2I = 2\int_0^{\pi} \log \sin x \, dx$$

$$\Rightarrow I = \int_0^{\pi} \log \sin x \, dx \qquad ...(3)$$

sin (π − x) = sin x

$$\therefore I = 2\int_0^{\frac{\pi}{2}} \log \sin x \, dx \qquad ...(4)$$

$$\Rightarrow I = 2\int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) dx = 2\int_0^{\frac{\pi}{2}} \log \cos x \, dx \qquad ...(5)$$

Adding (4) and (5), we obtain

$$2I = 2\int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x + \log 2 - \log 2) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log 2 \sin x \cos x - \log 2) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \sin 2x \, dx - \int_0^{\frac{\pi}{2}} \log 2 \, dx$$

Let 2x = t ⇒ 2dx = dt

When x = 0, t = 0 and when $x = \frac{\pi}{2}, t = \pi$

$$\therefore I = \frac{1}{2}\int_0^{\pi} \log \sin t \, dt - \frac{\pi}{2} \log 2$$

$$\Rightarrow I = \frac{1}{2} I - \frac{\pi}{2} \log 2$$

$$\Rightarrow \frac{I}{2} = -\frac{\pi}{2} \log 2$$

$$\Rightarrow I = -\pi \log 2$$

Question 17:

$$\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$$

Answer

$$\text{Let } I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx \qquad \ldots(1)$$

It is known that, $\left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$$

$$\Rightarrow 2I = \int_0^a 1\,dx$$

$$\Rightarrow 2I = [x]_0^a$$

$$\Rightarrow 2I = a$$

$$\Rightarrow I = \frac{a}{2}$$

Question 18:

$$\int_0^4 |x-1|dx$$

Answer

$$I = \int_0^4 |x-1|dx$$

It can be seen that, $(x - 1) \le 0$ when $0 \le x \le 1$ and $(x - 1) \ge 0$ when $1 \le x \le 4$

$$I=\int_0^1|x-1|dx+\int_1^4|x-1|dx \qquad \left(\int_a^b f(x)=\int_a^c f(x)+\int_c^b f(x)\right)$$

$$=\int_0^1-(x-1)dx+\int_1^4(x-1)dx$$

$$=\left[x-\frac{x^2}{2}\right]_0^1+\left[\frac{x^2}{2}-x\right]_1^4$$

$$=1-\frac{1}{2}+\frac{(4)^2}{2}-4-\frac{1}{2}+1$$

$$=1-\frac{1}{2}+8-4-\frac{1}{2}+1$$

$$=5$$

Question 19:

Show that $\int_0^a f(x)g(x)dx=2\int_0^a f(x)dx,$ if f and g are defined as $f(x)=f(a-x)$ and $g(x)+g(a-x)=4$

Answer

$$\text{Let } I=\int_0^a f(x)g(x)dx \qquad \ldots(1)$$

$$\Rightarrow I=\int_0^a f(a-x)g(a-x)dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^a f(x)g(a-x)dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^a\{f(x)g(x)+f(x)g(a-x)\}dx$$

$$\Rightarrow 2I=\int_0^a f(x)\{g(x)+g(a-x)\}dx$$

$$\Rightarrow 2I=\int_0^a f(x)\times 4\,dx \qquad [g(x)+g(a-x)=4]$$

$$\Rightarrow I=2\int_0^a f(x)dx$$

Question 20:

The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(x^3+x\cos x+\tan^5 x+1\right)dx$ is

A. 0

B. 2

C. π

D. 1

Answer

$$\text{Let } I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(x^3+x\cos x+\tan^5 x+1\right)dx$$

$$\Rightarrow I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}x^3dx+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos x+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\tan^5 x dx+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}1\cdot dx$$

It is known that if *f*(*x*) is an even function, then $\int_{-a}^{a}f(x)dx=2\int_{0}^{a}f(x)dx$ and if *f*(*x*) is an odd function, then $\int_{-a}^{a}f(x)dx=0$

$$I=0+0+0+2\int_{0}^{\frac{\pi}{2}}1\cdot dx$$

$$=2[x]_0^{\frac{\pi}{2}}$$

$$=\frac{2\pi}{2}$$

$$\pi=$$

Hence, the correct Answer is C.

Question 21:

The value of $\int_{0}^{\frac{\pi}{2}}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ is

A. 2

B. $\frac{3}{4}$

C. 0

D. -2

Answer

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log\left[\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right]dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\cos x}{4+3\sin x}\right)dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left\{\log\left(\frac{4+3\sin x}{4+3\cos x}\right) + \log\left(\frac{4+3\cos x}{4+3\sin x}\right)\right\}dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x} \times \frac{4+3\cos x}{4+3\sin x}\right)dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log 1\,dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 0\,dx$$

$$\Rightarrow I = 0$$

Hence, the correct Answer is C.