Chapter 6:APPLICATION OF DERIVATIVES

Ex-6.1

Q1. Find the rate of change of the area of a circle with respect to its radius r when

$$\frac{dA}{dr} = \frac{d}{dr}(\pi r^2) = 2\pi r.$$

A.1. (a) r = 3cm

When r = 3 cm,

$\frac{dA}{dr}$ = 2 ×π× 3 cm = 6π.

Thus, the area of the circle is changing at the rate of 6π cm $^{2}\!/_{5}$

(b) r = 4cm

whenr = 4cm,

$\frac{dA}{dr}$ = 2 ×π× 4cm = 8π.

Thus, the area of the circle is changing at the rate of 8π cm $^{2}\!/_{5}$

Q2. The volume of a cube is increasing at the rate of 8 cm3 /s. How fast is the surface area increasing when the length of an edge is 12 cm?

A.2.

Let x be the length of edge,v be the value and s be the surface area of the cube then,

y = x3.

and S = 6x2, where x is a fxn of time.

Now, $\frac{dY}{dF} = 8\,cm^{3}\!/_{5}$

$\Rightarrow \frac{d}{dt}$ (x3) = 8

$\Rightarrow \frac{dx^3}{dx}\cdot\frac{dx}{dt} = 8$ (by chain rule)

$\Rightarrow$ 3x2 $\frac{dx}{dx} = 8.$

$\Rightarrow \frac{dx}{dt} = \frac{8}{3x^2}.$

Now, $\frac{dS}{dt} = \frac{d}{dt}$ (bx2) = $\frac{d\left(6x^2\right)}{dx} - \frac{dx}{dx}$ = 12x $\frac{8}{3x^2} = \frac{32}{x}$ cm $^{2}\!/_{5}$

When x = 12 cm,

$\frac{dS}{dt} = \frac{32}{12} = \frac{8}{3}$ cm $^{2}\!/_{5}$

Q3. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

A.3.

Let 'r' cm be the radius of the circle which is afxn of time.

Then, $\frac{dy}{dt}$ = 8 3cm/s as it is increasing.

Now, the area A of the circle is A = πr2.

So, the rate at which the area of the circle change $\frac{dA}{dt} = \frac{d}{dt}$ πr2.

$= \frac{d}{dr}\pi r^2 \cdot \frac{dr}{dt}.$

= 2πr 3

= 6πr. cm $^{2}/_{5}$

When r = 10cm,

$\frac{dA}{dt}$ = 6.π× 10 = 60π cm $^{2}/_{5}$.

Q4. An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?

A.4.

Let 'x' cm be the length of edge of the cube which is a fx^n of time t then,

$\frac{dx}{dt}$ = 3cm/s as it is increasing.

Now, volume v of the cube is $v = x^3$.

∵ Rate of change of volume of the cube $\frac{dv}{dt} = \frac{dq^3}{dt}$.

$= \frac{dx^3}{dx}\frac{dx}{dt}$

$3x^2.3$

$9x^2$ cm $^{2}/_{5}$

When x = 10cm.

$\frac{dv}{dt}$ = 9 × $(10)^2$ = 900 cm $^{2}/_{5}$

Q5. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

A.5.

The area A of the circle with radius π is A = πr^2

Then, rate of change in area $\frac{dA}{dt} = \frac{d\dot{x}r^2}{dt} = \frac{dxr^2}{dr} \cdot \frac{dr}{dt}$.

$= 2\pi r \frac{dy}{dt}.$

∵ The wave moves at a rate 5cm/s we have,

$\frac{dr}{dF}$ = 5cm/s

So, $\frac{dA}{dt}$ =r. 5 = 10πr. cm $^{2}\!/_{5.}$

When r = 8 cm.

$\frac{dA}{dt}$ = 10.π.8 cm $^{2}\!/_{5}$ = 80 × cm $^{2}\!/_{5}$

Q6. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?

A.6.

The circumference C of the circle with radius r is C = 2πr.

Then, rate of change in circumference is $\frac{dC}{dt} = \frac{d}{dt} 2\pi r$ = $2\pi \frac{dx}{df}$.

∵ Radius of circle increases at rate 0.7 cm/s we get,

$$\frac{dy}{dt} = 0.7 \text{ cm/s}$$

So, $\frac{dC}{dt}$ = 2×. 0.7 cm/s = 1.4× cm/s.

Q7. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8cm and y = 6cm, find the rates of change of (a) the perimeter, and (b) the area of the rectangle.

A.7.

Since the length x is decreasing and the widthy is increasing with respect to time we have

$\frac{dx}{dt}$ = – 5cm/min and $\frac{dy}{dt}$ = A cm/min

(a) The perimeter P of a rectangle will be, P = 2(x + y)

∵ Rate of change of perimeter, $\frac{dP}{dt} = \frac{d^2}{dt}(x+y)$

$$= 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$

= 2(– 5 + 4)

= – 2 cm/min

(*b*) The area A of the rectangle is A = x. y.

∵ Rate of change of area is $\frac{dA}{dt} = \frac{d}{dt}(x \cdot y)$

$$= x\frac{dy}{dt} + \frac{dx}{dt} \cdot y.$$

= 4x– 5y

So, $\frac{dA}{dt}$ |x = 8ay = 6cm = 4(8) – 5(6) = 32 – 30 = 2 cm^2/min spherical balloon

Q8. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm

A.8. Let 'r' cm be the radius of volume V. measured Then,

$$V = \frac{4}{3}\pi r^3$$

Now, rate at which balloon is being inflated = 900cm $^{3}\!/_{5}$

$\Rightarrow \dfrac{dv}{dr} = 900\,\text{cm}\ ^{3}\!/_{5}$

$\Rightarrow \dfrac{d}{dt}\left(\dfrac{4}{3}\pi r^3\right)$ = 900 cm $^{3}\!/_{5.}$

$\Rightarrow \dfrac{d}{dr}\left(\dfrac{4}{3}\times r^3\right)\dfrac{dr}{dt} = 900$

$\dfrac{4}{3}\pi\times 3\times r^2\dfrac{dr}{dr}$ = 900.

$\Rightarrow \dfrac{dr}{dt} = \dfrac{900}{4\pi r^2}.$

When r = 15cm,

$\left.\dfrac{ds}{dt}\right|_{r=15} = \dfrac{900}{4\pi\times(15)^2} = \dfrac{900}{900\pi} = \dfrac{1}{\pi}$ cm/s.

$\because$ Radius of balloon increases by $\dfrac{1}{\pi}$ per second.

Q9. A balloon, which always remains spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm

A.9.

The volume v of a spherical balloon with radius r is $\text{V}.\dfrac{4}{3}\pi r^3$.

with respect its radius.

Then, the rate of change of volume $\left|\dfrac{d\text{V}}{dr} = \dfrac{d}{dr}\left(\dfrac{4}{3}\cdot\pi r^3\right)\right)$

$= \dfrac{4}{3}\pi\left(\dfrac{d}{dr}r^3\right)$

$= \dfrac{4}{3}\pi\times 3\times\pi^2$

$= 4\pi r^2$

When$x$ = 10 cm,

$\because \dfrac{d\text{V}}{dr}$ = $4\pi(10)^2 = 400\pi$ cm^3/cm

Q10. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

A.10.

Since, the bottom of ground is increasing with time t,

$\dfrac{dx}{dt}$ = 2cm/s

From fig, Δ ABC, by Pythagorastheorem

$AB^2 + BC^2 = AC^2$

$\Rightarrow x^2 + y^2 = 5^2$

$\Rightarrow x^2 + y^2 = 25$ ____ (1)

Differentiating eq^n (1) w. r. t. time t we get,

$$\frac{d}{dt}\left(x^2+y^2\right)=\frac{d}{dt}(25)$$

$$\Rightarrow 2x\frac{dx}{dt}+2y\frac{dy}{dt}=0.$$

$$\Rightarrow 2x\times 2+2y\frac{2y}{dy}=0$$

$$\Rightarrow \frac{dy}{dt}=-\frac{2x}{y}.\ m/s$$

When x = 4m, the rate at which its height on the wall decreases is

$$\frac{dy}{dt}=-\frac{2\times 4}{3} \qquad \begin{cases}\therefore 4^2+y^2=5^2\\ \Rightarrow y^2=25-16\\ \Rightarrow y\sqrt{9} \qquad (\text{Length cant'be negetive})\\ \Rightarrow y=3\end{cases}$$

$$\Rightarrow \frac{dy}{dt}=-\frac{8}{3}\text{ room}$$

Q11. A particle moves along the curve 6y = x3 +2. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate

A.11.

Given eq^n of the curve is 6y = x^3 + 2.______ (1)

When*y* coordinate change s 8 times as fast as *x*-coordinate

$\frac{dy}{dx}$ = 8 _____ (2)

Now, differentiating eq^n (1) wrt.*x* we get,

$$\frac{d}{dx}(6y)=\frac{d}{dx}\left(x^3+2\right)$$

$$\Rightarrow 6\frac{dy}{dx}=3x^2+0.$$

$\Rightarrow$ 6 × 8 = 3x^2 (using eq^n(2))

$$\Rightarrow \quad x^2=\frac{48}{3}=16$$

$$\Rightarrow +\sqrt{x}=\pm\sqrt{16}$$

$\Rightarrow x = \pm$ 4.

When x = 4, we have, 6y = 4^3 + 2 = 64 + 2 + 66

$$\Rightarrow y=\frac{66}{6}\, y = 11.$$

And when x = – 4, we have, 6y = (– 4)3 + 2 = – 64 + 2 = –62

$$\Rightarrow y=\frac{62}{6}=\frac{31}{3}$$

$\because$ The tequired point s are (4, 11) and $\left(-4,\frac{31}{3}\right)$

Q12. The radius of an air bubble is increasing at the rate of 1 2 cm/s. At what rate is the volume of the bubble increasing when the radius is 1 cm?

A.12.

Let x be the radius of the bubble with volume .V. then,

$$\frac{dr}{dt} = \frac{1}{2}\text{ cm/s}$$

and$V = \frac{4}{3}\pi n^3$

Rate of change of volume $\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi^3\right) = \frac{4}{3}\pi\frac{d}{dt^4}r^3$

$= \frac{4}{3}\pi \times 3r^2\frac{dr}{dt}.$

$= 4\pi r^2 \times \frac{1}{2}$

$= 2\pi r^2.$

$\because \left.\frac{dv}{dt}\right|_{r=cm} = 2x(1)^2 = 2\pi.\text{ cm}^{3}\!/_{5}.$

Q13. A balloon, which always remains spherical, has a variable diameter 3 (2 1) 2 x + . Find the rate of change of its volume with respect to x.

A.13.

Given, diameter of the spherical balloon = $\frac{3}{2}(2x + 1)$

So, radius of the spherical $r = \frac{1}{2}\times\frac{3}{2}(2x+1)$

$= \frac{3}{4}(2x+1)$

Then, volume of the spherical V = $\frac{4}{9}\pi r^3,$

$= \frac{4}{3}\pi\times\left[\frac{3}{4}\left(\frac{3}{(2x+1)}\right)\right]^3$

$= \frac{9\pi}{16}(2x+1)^3.$

$\because$Rate of change of volume wrt.to$x$, $\frac{dV}{dx} = \frac{d}{dx}\left[\frac{\pi}{16}(2x+1)^3\right]$

$= \frac{9\pi}{16}\times 3\times(2x+1)^2\cdot\frac{d}{dx}(2x+1)$

$= \frac{27\pi}{16}(2x+1)^2\times 2 = \frac{27\pi}{8}(2x+1)^2.$

Q14. Sand is pouring from a pipe at the rate of 12 cm³ /s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

A.14.

Let r cm and h cm be the radius and the height of the cone. Then,

$h = \frac{1}{6}r. \Rightarrow$ H = 6h

So, volume, V of the cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(6h)^2 \times h.$ $\frac{4d}{dt}.$

= 12 $\times h^3$

Rate of change of volume of the cone wrt the height is

$\frac{dV}{dh} = \frac{d}{dh}\left(12\pi h^3\right)$ = 12 $\times\pi\times$ 3 $\times h^2$.

As the sand is pouring from the pipe at rate of 12cm $^3/_{s.}$

we have

$$\frac{dV}{dt} = 12$$

$$\Rightarrow \frac{dv}{dh} \times \frac{dh}{dt} = 12$$

$$\Rightarrow 36\pi h^2 \frac{dh}{dt} = 12.$$

$$\Rightarrow \quad \frac{dh}{dt} = \frac{12}{36\pi h^2} = \frac{1}{3\pi h^2}.$$

$$\therefore \left.\frac{dh}{dt}\right|_{h=4\text{ cm}} = \frac{1}{3\pi \times (4)^2} = \frac{1}{48\pi}.$$

Hence, the height is increasing at the rate of $\frac{1}{48x}$ cm/s.

Q15. The total cost C (x) in Rupees associated with the production of x units of an item is given by C (x) = 0.007x^3 – 0.003x^2 + 15x + 4000. Find the marginal cost when 17 units are produced.

A.15.

Given, c(x) = 0.007 x^3– 0.003x^2 + 15x + 400.

Since the marginal cost is the rate of change of total cost wrt the output we have,

Marginal cost, MC, = $\frac{dC}{dx}(x)$

= 0.007 × 3x^2– 0.003 × 2x + 15.

When x = 17,

Then, MC = 0.007 ×2. (17)^2– 0.003 × 2(17) + 15.

= 6.069 – 0.102 + 15.

= 20.967

Hence, the required marginal cost = ` 20, 97.

Q16. The total revenue in Rupees received from the sale of x units of a product is given by R (x) = 13x2 + 26x + 15. Find the marginal revenue when x = 7.

A.16.

. Given, R(x) = 13x^2 + 26x + 15.

Marginal revenue is the rate of change of total revenue with respect to the number of units sold Marginal revenue (MR) = $\frac{dR(x)}{dx}$

$$= \frac{d}{d}\left(13x^2 + 26x + 15\right)$$

= 13 × 2x + 26

= 26x + 26

When x = 7

MR = 26 × 7 + 26 = 182 + 26 = 208.

Hence, the required marginal reverse = ` 208.

Choose the correct answer for questions 17 and 18.

Q17. The rate of change of the area of a circle with respect to its radius r at r = 6 cm is

(A) 10π (B) 12π (C) 8π (D) 11π

A.17.

The area A of the isle with radius r is given by with respect to radius r A = πr^2.

Then, rate of change of area of the circle $\frac{d \cdot A}{dx} = \frac{d\pi r^2}{dr}$

= $2\pi r$.

When r = 6 cm

$$\frac{dA}{dt} = 2\pi \times 6 = 12\pi.$$

∵ option (B) is correct.

Q18. **The total revenue in Rupees received from the sale of x units of a product is given by R(x) = 3x2 + 36x + 5. The marginal revenue, when x = 15 is**

(A) 116 (B) 96 (C) 90 (D) 126

A.18.

Given, R(x) = 3x^2 + 36x + 5.

Marginal revenue, $\frac{d}{dx}R(x) = \frac{d}{dx}\left(3x^2 + 36x + 5\right)$

= 3 × 2x + 36

= 6x + 36

When x = 15.

$\frac{d}{dx}R(x) = 6 \times 15 + 36$ = 90 + 36 = 126

∵Option (D) is correct.