

## Exercise 5.4

**Question 1:**

Differentiate the following w.r.t.  $x$ :

$$\frac{e^x}{\sin x}$$

Answer

$$\text{Let } y = \frac{e^x}{\sin x}$$

By using the quotient rule, we obtain

$$\begin{aligned}\frac{dy}{dx} &= \frac{\sin x \frac{d}{dx}(e^x) - e^x \frac{d}{dx}(\sin x)}{\sin^2 x} \\ &= \frac{\sin x \cdot (e^x) - e^x \cdot (\cos x)}{\sin^2 x} \\ &= \frac{e^x (\sin x - \cos x)}{\sin^2 x}, x \neq n\pi, n \in \mathbf{Z}\end{aligned}$$

**Question 2:**

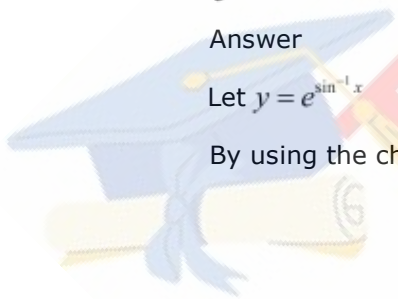
Differentiate the following w.r.t.  $x$ :

$$e^{\sin^{-1} x}$$

Answer

$$\text{Let } y = e^{\sin^{-1} x}$$

By using the chain rule, we obtain



$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} \left( e^{\sin^{-1} x} \right) \\
 \Rightarrow \frac{dy}{dx} &= e^{\sin^{-1} x} \cdot \frac{d}{dx} (\sin^{-1} x) \\
 &= e^{\sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}} \\
 &= \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} \\
 \therefore \frac{dy}{dx} &= \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}, x \in (-1, 1)
 \end{aligned}$$

**Question 2:**

Show that the function given by  $f(x) = e^{2x}$  is strictly increasing on  $\mathbf{R}$ .

Answer

Let  $x_1$  and  $x_2$  be any two numbers in  $\mathbf{R}$ .

Then, we have:

$$x_1 < x_2 \Rightarrow 2x_1 < 2x_2 \Rightarrow e^{2x_1} < e^{2x_2} \Rightarrow f(x_1) < f(x_2)$$

Hence,  $f$  is strictly increasing on  $\mathbf{R}$ .

**Question 3:**

Differentiate the following w.r.t.  $x$ :

$$e^{x^3}$$

Answer

$$\text{Let } y = e^{x^3}$$

By using the chain rule, we obtain

$$\frac{dy}{dx} = \frac{d}{dx} (e^{x^3}) = e^{x^3} \cdot \frac{d}{dx} (x^3) = e^{x^3} \cdot 3x^2 = 3x^2 e^{x^3}$$

**Question 4:**

Differentiate the following w.r.t.  $x$ :

$$\sin(\tan^{-1} e^{-x})$$

Answer

$$\text{Let } y = \sin(\tan^{-1} e^{-x})$$

By using the chain rule, we obtain

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [\sin(\tan^{-1} e^{-x})] \\&= \cos(\tan^{-1} e^{-x}) \cdot \frac{d}{dx} (\tan^{-1} e^{-x}) \\&= \cos(\tan^{-1} e^{-x}) \cdot \frac{1}{1+(e^{-x})^2} \cdot \frac{d}{dx} (e^{-x}) \\&= \frac{\cos(\tan^{-1} e^{-x})}{1+e^{-2x}} \cdot e^{-x} \cdot \frac{d}{dx} (-x) \\&= \frac{e^{-x} \cos(\tan^{-1} e^{-x})}{1+e^{-2x}} \times (-1) \\&= \frac{-e^{-x} \cos(\tan^{-1} e^{-x})}{1+e^{-2x}}\end{aligned}$$

**Question 5:**

Differentiate the following w.r.t.  $x$ :

$$\log(\cos e^x)$$

Answer

$$\text{Let } y = \log(\cos e^x)$$

By using the chain rule, we obtain

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [\log(\cos e^x)] \\&= \frac{1}{\cos e^x} \cdot \frac{d}{dx} (\cos e^x) \\&= \frac{1}{\cos e^x} \cdot (-\sin e^x) \cdot \frac{d}{dx} (e^x) \\&= \frac{-\sin e^x}{\cos e^x} \cdot e^x \\&= -e^x \tan e^x, e^x \neq (2n+1)\frac{\pi}{2}, n \in \mathbf{N}\end{aligned}$$

**Question 6:**

Differentiate the following w.r.t.  $x$ :

$$e^x + e^{x^2} + \dots + e^{x^5}$$

Answer

$$\begin{aligned} & \frac{d}{dx}(e^x + e^{x^2} + \dots + e^{x^5}) \\ &= \frac{d}{dx}(e^x) + \frac{d}{dx}(e^{x^2}) + \frac{d}{dx}(e^{x^3}) + \frac{d}{dx}(e^{x^4}) + \frac{d}{dx}(e^{x^5}) \\ &= e^x + \left[ e^{x^2} \times \frac{d}{dx}(x^2) \right] + \left[ e^{x^3} \cdot \frac{d}{dx}(x^3) \right] + \left[ e^{x^4} \cdot \frac{d}{dx}(x^4) \right] + \left[ e^{x^5} \cdot \frac{d}{dx}(x^5) \right] \\ &= e^x + (e^{x^2} \times 2x) + (e^{x^3} \times 3x^2) + (e^{x^4} \times 4x^3) + (e^{x^5} \times 5x^4) \\ &= e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5} \end{aligned}$$

**Question 7:**

Differentiate the following w.r.t.  $x$ :

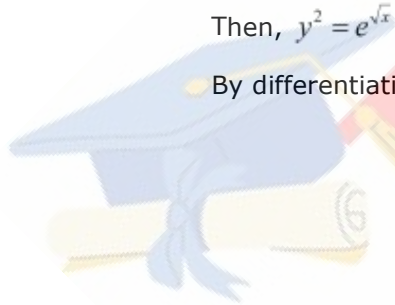
$$\sqrt{e^{\sqrt{x}}}, x > 0$$

Answer

$$\text{Let } y = \sqrt{e^{\sqrt{x}}}$$

$$\text{Then, } y^2 = e^{\sqrt{x}}$$

By differentiating this relationship with respect to  $x$ , we obtain



$$y^2 = e^{\sqrt{x}}$$

$$\Rightarrow 2y \frac{dy}{dx} = e^{\sqrt{x}} \frac{d}{dx}(\sqrt{x}) \quad [\text{By applying the chain rule}]$$

$$\Rightarrow 2y \frac{dy}{dx} = e^{\sqrt{x}} \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4y\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4\sqrt{e^{\sqrt{x}}}\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4\sqrt{x}e^{\sqrt{x}}}, x > 0$$

**Question 8:**

Differentiate the following w.r.t.  $x$ :

$$\log(\log x), x > 1$$

Answer

$$\text{Let } y = \log(\log x)$$

By using the chain rule, we obtain

$$\frac{dy}{dx} = \frac{d}{dx}[\log(\log x)]$$

$$= \frac{1}{\log x} \cdot \frac{d}{dx}(\log x)$$

$$= \frac{1}{\log x} \cdot \frac{1}{x}$$

$$= \frac{1}{x \log x}, x > 1$$

**Question 9:**

Differentiate the following w.r.t.  $x$ :

$$\frac{\cos x}{\log x}, x > 0$$

Answer

$$\text{Let } y = \frac{\cos x}{\log x}$$

By using the quotient rule, we obtain

$$\begin{aligned}\frac{dy}{dx} &= \frac{\frac{d}{dx}(\cos x) \times \log x - \cos x \times \frac{d}{dx}(\log x)}{(\log x)^2} \\ &= \frac{-\sin x \log x - \cos x \times \frac{1}{x}}{(\log x)^2} \\ &= \frac{-[x \log x \cdot \sin x + \cos x]}{x(\log x)^2}, x > 0\end{aligned}$$

**Question 10:**

Differentiate the following w.r.t.  $x$ :

$$\cos(\log x + e^x), x > 0$$

Answer

$$\text{Let } y = \cos(\log x + e^x)$$

By using the chain rule, we obtain

$$\begin{aligned}\frac{dy}{dx} &= -\sin(\log x + e^x) \cdot \frac{d}{dx}(\log x + e^x) \\ &= -\sin(\log x + e^x) \cdot \left[ \frac{d}{dx}(\log x) + \frac{d}{dx}(e^x) \right] \\ &= -\sin(\log x + e^x) \cdot \left( \frac{1}{x} + e^x \right) \\ &= -\left( \frac{1}{x} + e^x \right) \sin(\log x + e^x), x > 0\end{aligned}$$