www.toppersguru.com

Exrcise - 5·3

Find $\frac{dy}{dx}$ in the following:

Q1. $2x+3y=\sin x$

A.1. Given, $2n + 3y =$

Differentiating w r t x we get,

$$\frac{d}{dx}(2x+3y)=\frac{d}{dx}\sin x$$

$$\Rightarrow 2+3\frac{dy}{dx}=\cos x.$$

$$\Rightarrow 3\frac{dy}{dx}=\cos x-2$$

$$\Rightarrow \frac{dy}{dx}=\frac{\cos x-2}{3}.$$

Toppersguru

Q2. $2x + 3y = \sin y$

A.2. Given, $2x + 3y = \sin y$.

Differentiating w r t x. we get,

$$\frac{d}{dx}(2x+3y) = \frac{d}{dx}\sin y$$

$$\Rightarrow 2 + 3\frac{dy}{dx} = \cos y\frac{dy}{dx}$$

$$\Rightarrow \cos y\frac{dy}{dx} - 3\frac{dy}{dx} = 2$$

$$\Rightarrow \frac{dy}{dx}(\cos y - 3) = 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\cos y - 3}$$

Q3. $ax + by^2 = \cos\ y$

A.3. Given, $ax + \mathrm{b}y^2 = \cos y$.

Differentiating w r t 'x' we get,

$$\frac{d}{dx}\left(ax + by^2\right) = \frac{dx}{dx}\cos y$$

$$\Rightarrow a + b\ 2y = -\sin y\frac{dy}{dx} + \sin y\frac{dy}{dx} = -a$$

$$\Rightarrow \frac{dy}{dx} = \frac{-9}{2by + \sin y}.$$

$$\Rightarrow 2\mathrm{by}\ \frac{dy}{dx}$$

Q4. $xy + y^2 = \tan x + y$

A.4. Given, $xy + y^2 = \tan x + y$ Differentiating w r t x we get,

$$\frac{d}{dx}\left(xy + y^2\right) = \frac{d}{dx}(\tan x + y)$$

$$\Rightarrow x\frac{dy}{dx} + y\frac{dx}{dx} + \frac{dy^2}{dx} = dx^2 x + \frac{dy}{dx}$$

$$\Rightarrow x\frac{dy}{dx} + 2y\frac{dy}{dx} - \frac{dy}{dx} = \text{sen}^2 x - y$$

$$\Rightarrow (x + 2y - 1)\frac{dy}{dx} = \sec^2 x - y$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin^2 x - y}{x + 2y - 1.}$$

Q5. $x^2 + xy + y^2 = 100$

A.5. Given, $x^2 + xy + y^2 = 100$.

Differentiating w r t 'x' we get,

$$\frac{d}{dx}\left(x^2 + xy + y^2\right) = \frac{d}{dx}(100)$$

$$\Rightarrow 2x + x\frac{dy}{dx} + y\frac{dx}{dx} + 2y\frac{dy}{dx} = 0.$$

$$\Rightarrow x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(2x + y)}{(x + 2y)}$$

Q6. $x^3 + x^2y + xy^2 + y^3 = 81$

A.6. Given, $x^3 + x^2y + xy^2 + y^3 = 81$.

toppersguru

Differentiating w r t 'x' we get,

$$\frac{d}{dx}\left(x^3 + x^2 y + xy^2 + y^3\right) = \frac{d(81)}{dx}$$

$$\Rightarrow \frac{dx^3}{dx} + \frac{d}{dx}x^2 y + \frac{d}{dx}xy^2 + \frac{d}{dx}y^3 = 0$$

$$\Rightarrow 3x^2 + x^2\frac{dy}{dx} + y\frac{dx^2}{dx} + \frac{xdy^2}{dx} + y^2\frac{dx}{dx} + 3y^2\frac{dy}{dx} = 0$$

$$\Rightarrow 3x^2 + x^2\frac{dy}{dx} + 2xy + 2xy\frac{dy}{dx} + y^2 + 3y^2\frac{dy}{dx} = 0.$$

$$\Rightarrow \left(x^2 + 2xy + 3y^2\right)\frac{dy}{dx} = -(3x^2 + 2xy + y^2)$$

$$\frac{dy}{dx} = \frac{-\left(3x^2 + 2xy + y^2\right)}{\left(x^2 + 2xy + 3y^2\right).}$$

Q7. $\sin^2 y + \cos xy = k$

A.7. Given, $\sin^2 y + \cos xy = \pi$

Differentiating w r t 'x' we get,

$$\frac{d}{dx}\left(\sin^2 y + \cos xy\right) = \frac{d}{dx}$$

$$\Rightarrow \frac{d}{dx}\sin^2 y + \frac{d}{dx}\cos xy = 0.$$

$$\Rightarrow 2\sin y\frac{d}{dx}(\sin y) + (-\sin xy)\frac{d}{dx}(xy) = 0.$$

$$\Rightarrow 2\sin y\cos y\frac{dy}{dx} - \sin xy\left[x\frac{dy}{dx} + y\right] = 0.$$

$$\Rightarrow 2\sin 2y\frac{dy}{dx} - x\sin xy\frac{dy}{dx} - y\sin xy = 0$$

$\{\because \sin 2x = 2\sin x \cos x\}$

$$\Rightarrow \frac{dy}{dx}[\sin 2y - x\sin xy] = y\sin xy.$$

$$\Rightarrow \frac{dxy}{dx} = \frac{y\sin xy}{\sin 2y - x\sin xy}.$$

Q8. $\sin^2 x + \cos^2 y = 1$

A.8. Given, $\sin^2 x + \cos^2 y = 1$.

Differentiating w r t 'x' we get,

$$\frac{d}{dx}(\sin^2 x + \cos^2 y) = \frac{d}{dx}1.$$

$$\Rightarrow \frac{d}{dx}\sin^2 x + \frac{d}{dx}\cos^2 y = 0$$

$$\Rightarrow 2\sin x\frac{d\sin x}{dx} + 2\cos y\frac{d\cos y}{dx} = 0$$

$$\Rightarrow 2\sin x \cos x + 2\cos y(-\sin y)\frac{dy}{dx} = 0$$

$$\Rightarrow \sin 2x - \sin 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \sin 2y\frac{dy}{dx} = \sin 2x \qquad \{\because \sin 2\theta = 2\sin\theta\cos\theta\}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}.$$

Q9. $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

A.9 Given, $y = \sin^{-1}\left(\frac{2x}{3+x^2}\right)$.

www.toppersguru.com

Let $x = \tan\theta$. $\Rightarrow \theta = \tan^{-1}x$

$$\Rightarrow y = \sin^{-1}\left[\frac{2\tan\theta}{1+\tan^2\theta}\right] = \sin^{-1}(\sin 2\theta) = 2\theta.$$

$$\Rightarrow y = 2\tan^{-1}x. \qquad \left\{\because \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}\right\}$$

Differentiating w r t 'x' we get,

$$\frac{dy}{dx} = 2\frac{d}{dx}\tan^{-1}x$$

$$= 2 \times \frac{1}{1+x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{1+x^2}.$$

Q10. $y = \tan^{-1}\left(\frac{2x - x^3}{1-3x^2}\right), -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

A.10. Given,

$$y = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right)$$

Let $x = \tan\theta$. $\Rightarrow \theta = \tan^{-1}x$

$$y = \tan^{-1}\left[\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right]$$

$$y = \tan^{-1}[\tan 3\theta] \qquad \left\{\because \tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right\}$$

$$y = 3\theta$$

$$y = 3\tan^{-1}x.$$

So, $\frac{dy}{dx} = 3\frac{d\tan^{-1}x}{dx}$

toppersguru

$$\Rightarrow \frac{dy}{dx} = \frac{3}{1+x^2}$$

Q11. $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), -0 < x < 1$

A.11. Given, $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Let $x = \tan\theta \Rightarrow = \tan^{-1}x$.

So, $y = \cos^{-1}\left[\frac{1-\tan^2\theta}{1+\tan^2\theta}\right]$.

$y \cos^{-1}(\cos 2\theta)$ $\left\{\therefore \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}\right\}$.

$y = 2\theta$

$y = 2\tan^{-1}x$

$$\therefore \frac{dy}{dx} = 2\frac{d\tan^{-1}x}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{1+x^2}.$$

Q12. $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), -0 < x < 1$

A.12. Given, $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Let $x = \tan\theta \Rightarrow = \tan^{-1}x$.

So, $y = \sin^{-1}\left[\frac{1-\tan^2\theta}{1+\tan^2\theta}\right]$

$y = \sin^{-1}(\cos 2\theta)$.

$\Rightarrow y = \sin^{-1}\left[\sin\left(\frac{\pi}{2} - 2\theta\right)\right]$

$\Rightarrow y = \frac{\pi}{2} - 2\theta$.

$\Rightarrow y = \frac{\pi}{2} - 2\tan^{-1}x$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2} - 2\tan^{-1}x\right)$

$= 0 - \frac{2}{1+x^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-2}{1+x^2}$.

Q13. $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), -1 < x < 1$

A.13. Given, $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$

Let $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

Se, $y = \cos^{-1}\left[\frac{2\tan\theta}{1+\tan^2\theta}\right]$

$\Rightarrow y = \cos^{-1}(\sin 2\theta)$

$y = \cos^{-1}\left[\cos\left(\frac{\pi}{2} - 2\theta\right)\right]$

$y = \frac{\pi}{2} - 2\theta$.

$\Rightarrow y = \frac{\pi}{2} - 2\tan^{-1}x$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2} - 2\tan^{-1}x\right)$$

$$= 0 - \frac{2}{1+x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2}{1+x^2}$$

Q14. $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

A.14. Given, $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$.

Let $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$.

Then, $y = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$

$= \sin^{-1}(2\sin\theta\cos\theta)$ $\{\because \cos^2\theta + \sin^2\theta = 1 \ \sin 2\theta = 2\sin\cos\theta)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta$

$\Rightarrow y = 2\sin^{-1}x$

$$\therefore \frac{dy}{dx} = 2\frac{d}{dx}\sin^{-1}x$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}.$$

Q15. $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$

A.15. Given, $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$

Let $x = \cos\theta$. $\Rightarrow \theta = \cos^{-1}x$

Then, $y = \sec^{-1}\left[\frac{1}{2\cos^2\theta - 1}\right]$

$= \sec^{-1}\left[\frac{1}{\cos 2\theta}\right]$ $\{\because \cos 2\theta = 2\cos^2\theta - 1\}$

$\sec^{-1}(\sec 2\theta)$

$y = 2\theta$

$y = 2\cos^{-1}x$.

$\therefore \frac{dy}{dx} = \frac{2d\cos^{-1}x}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$.