Exercise 12.1

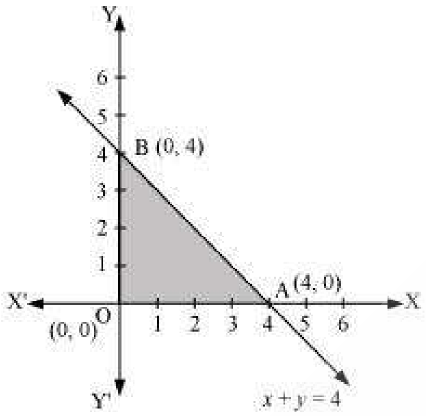
Question 1:

Maximise Z = 3*x* + 4*y*

Subject to the constraints: $x + y \leq 4, x \geq 0, y \geq 0.$

Answer

The feasible region determined by the constraints, $x + y \leq 4$, $x \geq 0$, $y \geq 0$, is as follows.



The corner points of the feasible region are O (0, 0), A (4, 0), and B (0, 4). The values of Z at these points are as follows.

Corner point	Z = 3*x* + 4*y*	
O(0, 0)	0	
A(4, 0)	12	
B(0, 4)	16	→ Maximum

Therefore, the maximum value of Z is 16 at the point B (0, 4).

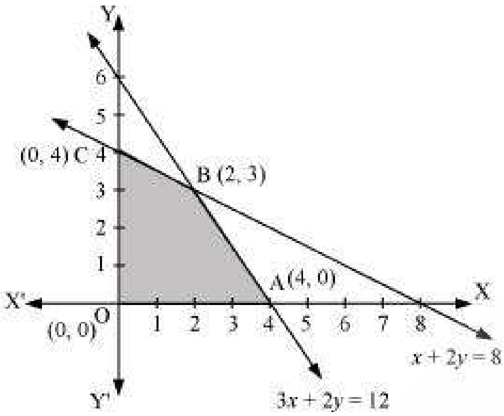
Question 2:

Minimise Z = −3*x* + 4*y*

subject to $x + 2y \leq 8,\ 3x + 2y \leq 12,\ x \geq 0,\ y \geq 0$.

Answer

The feasible region determined by the system of constraints, $x + 2y \leq 8,\ 3x + 2y \leq 12,$ $x \geq 0$, and $y \geq 0$, is as follows.



The corner points of the feasible region are O (0, 0), A (4, 0), B (2, 3), and C (0, 4).

The values of Z at these corner points are as follows.

Corner point	**Z = −3*x* + 4*y***	
0(0, 0)	0	
A(4, 0)	−12	→ Minimum
B(2, 3)	6	
C(0, 4)	16	

Therefore, the minimum value of Z is −12 at the point (4, 0).

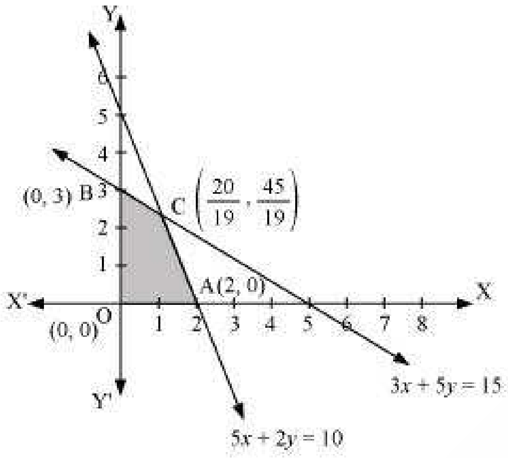
Question 3:

Maximise Z = $5x + 3y$

subject to $3x + 5y \leq 15,\ 5x + 2y \leq 10,\ x \geq 0,\ y \geq 0$.

Answer

The feasible region determined by the system of constraints, $3x + 5y \leq 15$,
$5x + 2y \leq 10$, $x \geq 0$, and $y \geq 0$, are as follows.



The corner points of the feasible region are O (0, 0), A (2, 0), B (0, 3), and $C\left(\frac{20}{19}, \frac{45}{19}\right)$.

The values of Z at these corner points are as follows.

Corner point	Z = 5*x* + 3*y*	
0(0, 0)	0	
A(2, 0)	10	
B(0, 3)	9	
$C\left(\frac{20}{19}, \frac{45}{19}\right)$	$\frac{235}{19}$	→ Maximum

Therefore, the maximum value of Z is $\frac{235}{19}$ at the point $\left(\frac{20}{19}, \frac{45}{19}\right)$.

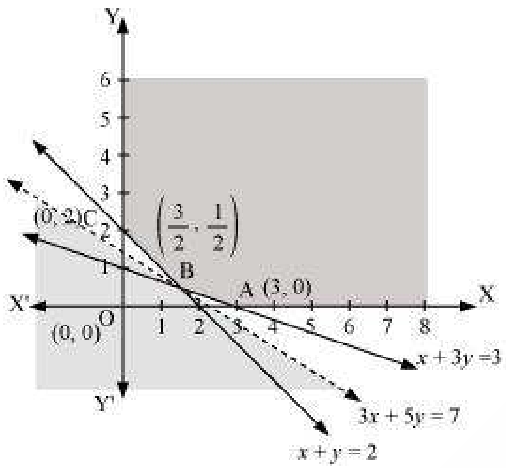
Question 4:

Minimise Z = 3*x* + 5*y*

such that $x+3y \geq 3,\ x+y \geq 2,\ x, y \geq 0$.

Answer

The feasible region determined by the system of constraints, $x+3y \geq 3, x+y \geq 2$, and *x*, *y* ≥ 0, is as follows.



It can be seen that the feasible region is unbounded.

The corner points of the feasible region are A (3, 0), $B\left(\frac{3}{2}, \frac{1}{2}\right)$, and C (0, 2).

The values of Z at these corner points are as follows.

Corner point	**Z = 3*x* + 5*y***	
A(3, 0)	9	
$B\left(\frac{3}{2}, \frac{1}{2}\right)$	7	→ Smallest
C(0, 2)	10	

As the feasible region is unbounded, therefore, 7 may or may not be the minimum value of Z.

For this, we draw the graph of the inequality, $3x + 5y < 7$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $3x + 5y < 7$ Therefore, the minimum value of Z is 7 at $\left(\frac{3}{2}, \frac{1}{2}\right)$.

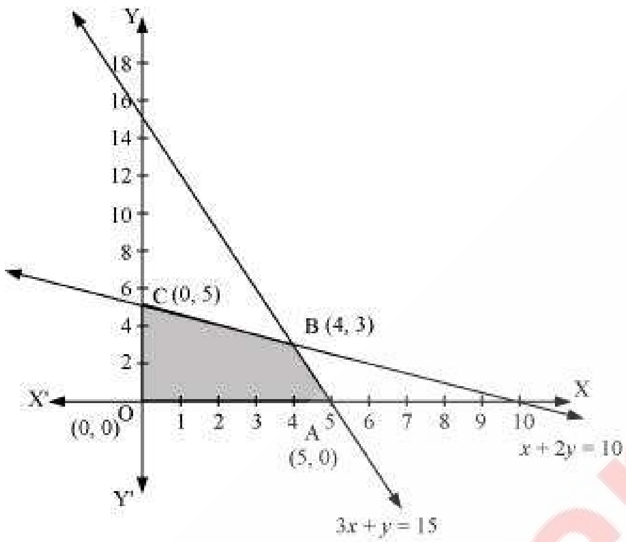
Question 5:

Maximise $Z = 3x + 2y$

subject to $x+2y\le 10, 3x+y\le 15, x, y\ge 0$.

Answer

The feasible region determined by the constraints, $x + 2y \le 10$, $3x + y \le 15$, $x \ge 0$, and $y \ge 0$, is as follows.



The corner points of the feasible region are A (5, 0), B (4, 3), and C (0, 5).

The values of Z at these corner points are as follows.

Corner point	**Z = 3*x* + 2*y***	
A(5, 0)	15	
B(4, 3)	18	→ Maximum
C(0, 5)	10	

Therefore, the maximum value of Z is 18 at the point (4, 3).

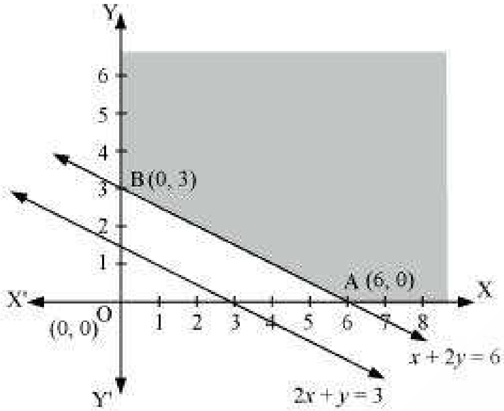
Question 6:

Minimise Z = $x + 2y$

subject to $2x + y \ge 3,\ x + 2y \ge 6,\ x, y \ge 0$.

Answer

The feasible region determined by the constraints, $2x + y \geq 3$, $x + 2y \geq 6$, $x \geq 0$, and $y \geq 0$, is as follows.



The corner points of the feasible region are A (6, 0) and B (0, 3).

The values of Z at these corner points are as follows.

Corner point	Z = $x + 2y$
A(6, 0)	6
B(0, 3)	6

It can be seen that the value of Z at points A and B is same. If we take any other point such as (2, 2) on line $x + 2y = 6$, then Z = 6

Thus, the minimum value of Z occurs for more than 2 points.

Therefore, the value of Z is minimum at every point on the line, $x + 2y = 6$

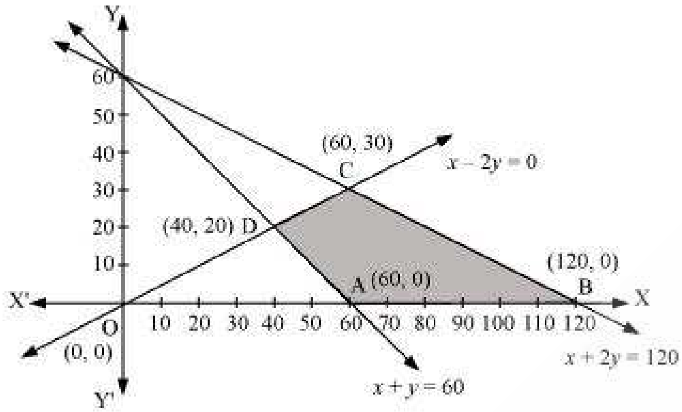
Question 7:

Minimise and Maximise $Z = 5x + 10y$

subject to $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x, y \geq 0$.

Answer

The feasible region determined by the constraints, $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, and $y \geq 0$, is as follows.



The corner points of the feasible region are A (60, 0), B (120, 0), C (60, 30), and D (40, 20).

The values of Z at these corner points are as follows.

Corner point	**Z = 5*x* + 10*y***	
A(60, 0)	300	→ Minimum
B(120, 0)	600	→ Maximum
C(60, 30)	600	→ Maximum
D(40, 20)	400	

The minimum value of Z is 300 at (60, 0) and the maximum value of Z is 600 at all the points on the line segment joining (120, 0) and (60, 30).

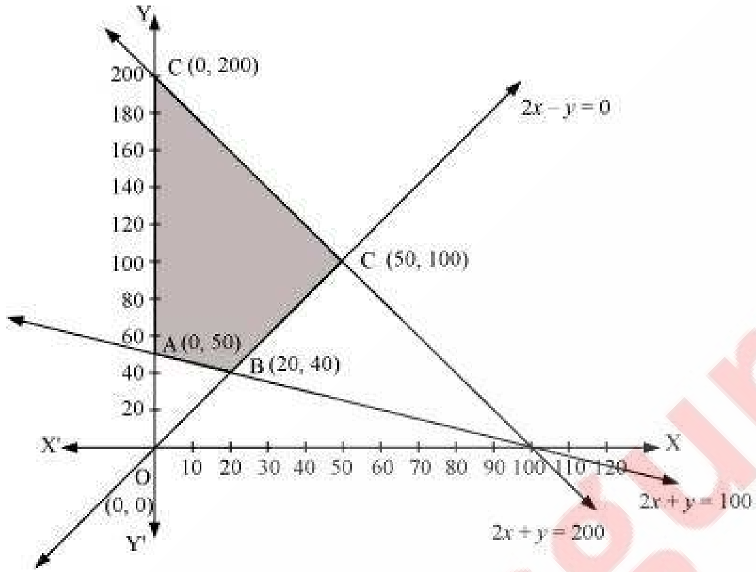
Question 8:

Minimise and Maximise Z = $x + 2y$

subject to $x+2y \geq 100,\ 2x-y \leq 0,\ 2x+y \leq 200;\ x, y \geq 0$.

Answer

The feasible region determined by the constraints, $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x \geq 0$, and $y \geq 0$, is as follows.



The corner points of the feasible region are A(0, 50), B(20, 40), C(50, 100), and D(0, 200).

The values of Z at these corner points are as follows.

Corner point	**Z = *x* + 2*y***	
A(0, 50)	100	→ Minimum
B(20, 40)	100	→ Minimum
C(50, 100)	250	
D(0, 200)	400	→ Maximum

The maximum value of Z is 400 at (0, 200) and the minimum value of Z is 100 at all the points on the line segment joining the points (0, 50) and (20, 40).

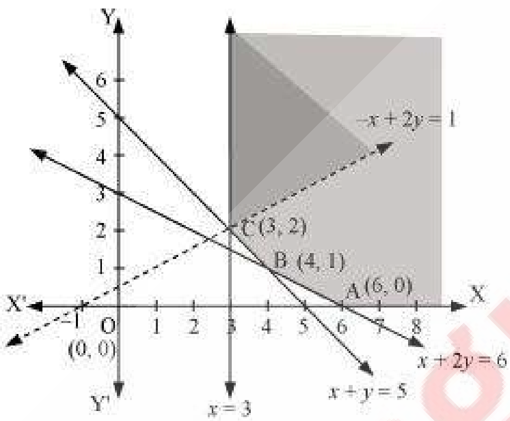
Question 9:

Maximise Z = − x + 2y, subject to the constraints:

$x \geq 3,\ x + y \geq 5,\ x + 2y \geq 6,\ y \geq 0$.

Answer

The feasible region determined by the constraints, $x \geq 3,\ x+y \geq 5,\ x+2y \geq 6, \text{ and } y \geq 0,$ is as follows.



It can be seen that the feasible region is unbounded.

The values of Z at corner points A (6, 0), B (4, 1), and C (3, 2) are as follows.

Corner point	**Z = −*x* + 2*y***
A(6, 0)	Z = − 6
B(4, 1)	Z = − 2
C(3, 2)	Z = 1

As the feasible region is unbounded, therefore, Z = 1 may or may not be the maximum value.

For this, we graph the inequality, $-x + 2y > 1$, and check whether the resulting half plane has points in common with the feasible region or not.

The resulting feasible region has points in common with the feasible region.

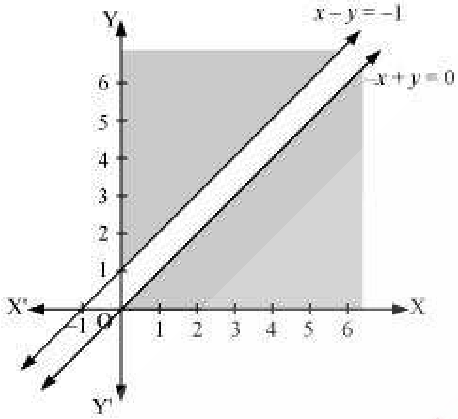
Therefore, Z = 1 is not the maximum value. Z has no maximum value.

Question 10:

Maximise Z = $x + y$, subject to $x - y \leq -1,\ -x + y \leq 0,\ x, y \geq 0$.

Answer

The region determined by the constraints, $x - y \le -1, -x + y \le 0, x, y \ge 0,$ is as follows.



There is no feasible region and thus, Z has no maximum value.