Exercise 11.2

Q1. Show that the three lines with direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}; \frac{4}{13}, \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.

A.1. Two lines with direction cosines l, m, n and l_2, m_2, n_2 are perpendicular to each other, if $l_1l_2 + m_1m_2 + n_1n_2 = 0$.

Now, for the 3 lines with direction cosine,

$$\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13} \text{ and } \frac{4}{13}, \frac{12}{13}, \frac{3}{13}$$

$$l_1l_2 + m_1m_2 + n_1n_2 = \frac{12}{13} \times \frac{4}{13} + \frac{(-3)}{13} \times \frac{12}{13} + \frac{(-4)}{13} \times \frac{3}{13}$$

$$= \frac{48}{169} - \frac{36}{169} - \frac{12}{169}$$

$$= 0$$

Hence, the lines are perpendicular.

For lines with direction cosines,

$$\frac{4}{13}, \frac{12}{13}, \frac{3}{13} \text{ and } \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$$

$$l_1l_2 + m_1m_2 + n_1n_2 = \frac{4}{13} \times \frac{3}{13} + \frac{12}{13} \times \frac{(-4)}{13} + \frac{3}{13} \times \frac{12}{13}$$

$$= \frac{12}{169} - \frac{48}{169} + \frac{36}{169}$$

$$= 0$$

Hence, these lines are perpendicular.

For the lines with direction cosines,

$$\frac{3}{13}, \frac{-4}{13}, \frac{12}{13} \text{ and } \frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}$$

$$l_1l_2 + m_1m_2 + n_1n_2 = \frac{3}{13} \times \frac{12}{13} + \frac{(-4)}{13} \times \frac{(-3)}{13} + \frac{12}{13} \times \frac{(-4)}{13}$$

$$= \frac{36 + 12 - 48}{169}$$

$$= 0$$

Hence, these lines are perpendicular.

Therefore, all the lines are perpendicular.

Q2. Show that the line through the points (1, −1, 2) (3, 4, −2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

A2. Let AB be the line joining the points $(1, -1, 2)$ and $(3, 4, -2)$ and CD be the line joining the point $(0, 3, 2)$ and $(3, 5, 6)$.

The direction ratios, a, b, c of AB are $(3-1),(4-(-1)),(-2-2)$
$=(2,5,-4)$

The direction ratios a_2, b_2, c_2 of CD are $(3-0),(5-3),(6-2)$
$=(3,2,4)$

AB and CD will be perpendicular to each other, if $a_1a_2+b_1b_2+c_1c_2=0$

$$=2\times3+5\times2+(-4)\times4$$
$$=6+10-16$$
$$=0$$

$\therefore$ Therefore, AB and CD are perpendicular to each other.

Q3. Show that the line through points (4, 7, 8), (2, 3, 4) is parallel to the line through the points (−1, −2, 1), (1, 2, 5).

A.3. Let AB be the line through the point $(4,7,8)$ and $(2,3,4)$ and CD be line through the point $(-1,-2,1)$ and $(1,2,5)$

Direction cosine, a_1, b_1, c_1 of AB are
$$=(2-4),(3-7),(4-8)$$
$$=(-2,-4,-4)$$

Direction cosine, a_2, b_2, c_2 of CD are

$$=(1-(-1)),(2-(-2)),(5-1)$$
$$=(2,4,4)$$

AB will be parallel to CD only

If

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$
$$\Rightarrow\frac{-2}{2}=\frac{-4}{4}=\frac{-4}{4}$$
$$\Rightarrow-1=-1=-1$$

Here, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Therefore, AB is parallel to CD.

Q4. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i}+2\hat{j}-2\hat{k}$

A.4. Given,

The line passes through the point $A(1,2,3)$.

Position vector of A,

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

Let $\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$

The line which passes through point $\vec{a}$ and parallel to $\vec{b}$ is given by,

$$\vec{r} = \vec{a} + \lambda\vec{b}$$
$$= \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} + 2\hat{j} - 2\hat{k}\right)$$, where λ is constant

Q5. Find the equation of the line in vector and in Cartesian form that passes through the point with position vector $2\hat{i} + \hat{j} + 4\hat{k}$ and is in the direction $\hat{i} + 2\hat{j} - \hat{k}$

A5. The line passes through the point with position vector, $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k} - - - - - (1)$

The given vector: $\vec{b} = \hat{i} + 2\hat{j} - \hat{k} - - - - - (2)$

The line which passes through a point with position vector $\vec{a}$ and parallel to $\vec{b}$ is given by,

$$\vec{r} = \vec{a} + \lambda\vec{b}$$
$$\vec{r} = 2\hat{i} - \hat{j} + 4\hat{k} + \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right)$$

$\therefore$ This is required equation of the line in vector form.

Now,

Let
$$\vec{r} = x\hat{i} - y\hat{j} + z\hat{k}$$
$$\Rightarrow x\hat{i} - y\hat{j} + z\hat{k} = (\lambda + 2)\hat{i} + (2\lambda - 1)\hat{j} + (-\lambda + 4)\hat{k}$$

Comparing the coefficient to eliminate λ,

$$x = \lambda + 2, x_1 = 2, a = 1$$
$$y = 2\lambda - 1, y_1 = -1, b = 2$$
$$z = -\lambda + 4, z_1 = 4, c = -1$$

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$$

Q6. Find the Cartesian equation of the line which passes through the point (−2, 4, −5) and parallel to the line given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$

A.6. Given,

The point $(-2,4,-5)$.

The Cartesian equation of a line through a point (x_1,y_1,z_1) and having direction ratios a, b, c is

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

Now, given that

$\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ is parallel

to point $(-2,4,-5)$

Here, the point (x_1,y_1,z_1) is $(-2,4,-5)$ and the direction ratio is given by $a=3, b=5, c=6$

$\therefore$ The required Cartesian equation is

$$\frac{x-(-2)}{3}=\frac{y-4}{5}=\frac{z-(-5)}{6}$$

$$\Rightarrow \frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$$

Q7. The Cartesian equation of a line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$ **Write its vector form.**

A.7. Given,

Cartesian equation,

$$\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$$

The given line passes through the point $(5,-4,6)$

i.e. position vector of $\vec{a}=5\hat{i}-4\hat{j}+6\hat{k}$

Direction ratio are 3, 7 and 2.

Thus, the required line passes through the point $(5,-4,6)$ and is parallel to the vector $3\hat{i}+7\hat{j}+2\hat{k}$.

Let $\vec{r}$ be the position vector of any point on the line, then the vector equation of the line is given by,

$\vec{r} = \left(5\hat{i} - 4\hat{j} + 6\hat{k}\right) + \lambda\left(3\hat{i} + 7\hat{j} + 2\hat{k}\right)$

Q8. Find the vector and Cartesian equations of the line that passes through the origin and (5, −2, 3).

A.8. The required line passes through the origin. Therefore, its position vector is given by,

$\vec{a} = 0$.........(1)

The direction ratios of the line through origin and $(5,\ -2,\ 3)$ are

$(5 - 0) = 5,\ (-2 - 0) = -2,\ (3 - 0) = 3$

The line is parallel to the vector given by the equation, $\vec{b} = 5\hat{i} - 2j + 3k$

The equation of the line in vector form through a point with position vector $\vec{a}$ and parallel to $\vec{b}$ is, $\vec{r} = \vec{a} + \lambda\vec{b}, \lambda \in R$

$\Rightarrow \vec{r} = \vec{0} + \lambda\left(5\hat{i} - 2j + 3k\right)$

$\Rightarrow \vec{r} = \lambda\left(5\hat{i} - 2j + 3k\right)$

The equation of the line through the point (x_1, y_1, z_1) and direction ratios a, b, c is given by,

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Therefore, the equation of the required line in the Cartesian form is

$$\frac{x - 0}{5} = \frac{y - 0}{-2} = \frac{z - 0}{3}$$

$$\Rightarrow \frac{x}{5} = \frac{y}{-2} = \frac{z}{3}$$

Q9. Find vector and Cartesian equations of the line that passes through the points $(3,\ -2,\ -5)$, $(3,\ -2,\ 6)$.

A.9.

Let the line passing through the points, P (3, −2, −5) and Q (3, −2, 6), be PQ.

Since PQ passes through P (3, −2, −5), its position vector is given by,

$\vec{a} = 3\hat{i} - 2j - 5k$

The direction ratios of PQ are given by,

$(3 - 3) = 0,\ (-2 + 2) = 0,\ (6 + 5) = 11$

The equation of the vector in the direction of PQ is

$\vec{b} = 0.\hat{i} - 0.j + 11k = 11k$

The equation of PQ in vector form is given by, $\vec{r} = \vec{a} + \lambda\vec{b}, \lambda \in R$

$\Rightarrow \vec{r} = \left(5\hat{i} - 2j - 5k\right) + 11\lambda k$

The equation of PQ in Cartesian form is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$ i.e,

$$\frac{x - 3}{0} = \frac{y + 2}{0} = \frac{z + 5}{11}$$

Q10. Find the angle between the following pairs of lines:

$(i)\vec{r} = 2\hat{i} - 5j + k + \lambda\left(3\hat{i} - 2j + 6k\right) \,\&\, \vec{r} = 7\hat{i} - 6k + \mu\left(\hat{i} + 2j + 2k\right)$

$(ii)\vec{r} = 3\hat{i} + j - 2k + \lambda\left(\hat{i} - j - 2k\right) \,\&\, \vec{r} = 2\hat{i} - j - 56k + \mu\left(3i - 5j - 4k\right)$

A.10.

(i) Let Q be the angle between the given lines.

The angle between the given pairs of lines is given by, $\cos Q = \left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$

The given lines are parallel to the vectors, $\vec{b}_1 = 3\hat{i} + 2j + 6k \,\&\, \vec{b}_2 = \hat{i} + 2j + 2k$, respectively.

$\therefore |\vec{b}_1| = \sqrt{3^2 + 2^2 + 6^2} = 7$

$|\vec{b}_2| = \sqrt{(1)^2 + (2)^2 + (2)^2} = 3$

$\vec{b}_1.\vec{b}_2 = \left(3\hat{i} + 2j + 6k\right).\left(\hat{i} + 2j + 2k\right)$

$= 3\times1 + 2\times2 + 6\times2$

$= 3 + 4 + 12$

$= 19$

$\Rightarrow \cos Q = \frac{19}{7\times3}$

$\Rightarrow Q = \cos^{-1}\left(\frac{19}{21}\right)$

(ii) The given lines are parallel to the vectors, $\vec{b}_1 = \hat{i} - j - 2k \,\&\, \vec{b}_2 = 3\hat{i} - 5j - 4k$ respectively

$\vec{b}_1 = \hat{i} - j - 2k \,\&\, \vec{b}_2 = 3\hat{i} - 5j - 4k$

$\therefore |\vec{b}_1| = \sqrt{(1)^2 + (-1)^2 + (-2)^2} = \sqrt{6}$

$|\vec{b}_2| = \sqrt{(3)^2 + (-5)^2 + (-4)^2} = \sqrt{50} = 5\sqrt{2}$

$\vec{b}_1.\vec{b}_2 = \left(\hat{i} - j - 2k\right).\left(3\hat{i} - 5j - 4k\right)$

$= 1.3 - 1(-5) - 2(-4)$

$= 3 + 5 + 8$

$= 16$

$\cos Q = \left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$

$\Rightarrow \cos Q = \frac{16}{\sqrt{6}.5\sqrt{2}} = \frac{16}{\sqrt{2}.\sqrt{3}.5\sqrt{2}} = \frac{16}{10\sqrt{3}}$

$\Rightarrow \cos Q = \frac{8}{5\sqrt{3}}$

$\Rightarrow Q = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$

Q11. Find the angle between the following pair of lines

$$(i)\ \frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3}\ \&\ \frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$$

$$(ii)\ \frac{x}{y}=\frac{y}{2}=\frac{z}{1}\ \&\ \frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$$

A.11.

(i) Let $\vec{b}_1$ and $\vec{b}_2$ be the vectors parallel to the pair of lines, $\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3}\ \&\ \frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$, respectively.

$$\vec{b}_1 = 2\hat{i}+5j-3k\ \&\ \vec{b}_2 = -\hat{i}+8j+4k$$

$$\therefore \left|\vec{b}_1\right| = \sqrt{(2)^2+(5)^2+(-3)^2} = \sqrt{38}$$

$$\left|\vec{b}_2\right| = \sqrt{(-1)^2+(8)^2+(4)^2} = \sqrt{81} = 9$$

$$\vec{b}_1.\vec{b}_2 = \left(2\hat{i}+5j-3k\right).\left(-\hat{i}+8j+4k\right)$$

$$= 2(-1)+5\times 8+(-3).4$$

$$= -2+40-12$$

$$= 26$$

The angle, Q, between the given pair of lines is given by the relation,

$$\cos Q = \left|\frac{\vec{b}_1.\vec{b}_2}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}\right|$$

$$\Rightarrow \cos Q = \frac{26}{9\sqrt{38}}$$

$$\Rightarrow Q = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$$

(ii) Let $\vec{b}_1$, $\vec{b}_2$ be the vectors parallel to the given pair of lines, $\frac{x}{y}=\frac{y}{2}=\frac{z}{1}\ \&\ \frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$, respectively.

$$_1 = 2\hat{i}+2j+k\ \&\ \vec{b}_2 = 4\hat{i}+j+8k$$

$$\therefore \left|\vec{b}_1\right| = \sqrt{(2)^2+(2)^2+(1)^2} = \sqrt{9} = 3$$

$$\left|\vec{b}_2\right| = \sqrt{4^2+1^2+8^2} = \sqrt{81} = 9$$

$$\vec{b}_1.\vec{b}_2 = \left(2\hat{i}+2j+k\right).\left(4\hat{i}+j+8k\right)$$

$$= 2\times 4+2\times 1+1\times 8$$

$$= 8+2+8$$

$$= 18$$

If Q is the angle between the given pair of lines, then $\cos Q = \left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$

$$\Rightarrow \cos Q = \frac{18}{3\times 9} = \frac{2}{3}$$

$$\Rightarrow Q = \cos^{-1}\left(\frac{2}{3}\right)$$

Q12. Find the values of *p* so that the lines $\frac{1-x}{3} = \frac{7y-14}{2}p = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ **are at right angles.**

A.12. $\frac{1-x}{3} = \frac{7y-14}{2\rho} = \frac{z-3}{2}\ and\ \frac{7-7x}{3\rho} = \frac{y-5}{1} = \frac{6-z}{5}$

The standard form of a pair of Cartesian lines is;

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}\ and\ \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2} ------(1)$$

So,

$$\frac{-(x-1)}{3} = \frac{7(y-5)}{2\rho} = \frac{z-3}{2}\ and\ \frac{-7(x-1)}{3\rho} = \frac{y-5}{1} = \frac{-(z-6)}{5}$$

$$\frac{x-1}{-3} = \frac{y-2}{2\rho/7} = \frac{z-3}{2}\ and\ \frac{x-1}{-3\rho/7} = \frac{y-5}{1} = \frac{z-6}{-5} ------(2)$$

Comparing (1) and (2) we get

$$a_1 = -3, b_1 = \frac{2\rho}{7}, c_1 = 2$$

$$a_2 = \frac{-3\rho}{7}, b_2 = 1, c_2 = -5$$

Now, both the lines are at right angles

So, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\Rightarrow (-3)\times\frac{(-3\rho)}{7} + \frac{2\rho}{7}\times 1 + 2\times(-5) = 0$$

$$\Rightarrow \frac{9\rho}{7} + \frac{2\rho}{7} + (-10) = 0$$

$$\Rightarrow \frac{9\rho + 2\rho}{7} = 10$$

$$\Rightarrow 11\rho = 70$$

$$\rho = \frac{70}{11}$$

$\therefore$ The value of ρ is $\dfrac{70}{11}$

Q13. Show that the lines $\dfrac{x-5}{7}=\dfrac{y+2}{-5}=\dfrac{z}{1}$ and $\dfrac{x}{1}=\dfrac{y}{2}=\dfrac{z}{3}$ **are perpendicular to each other.**

A.13. $\dfrac{x-5}{7}=\dfrac{y+2}{-5}=\dfrac{z}{1}$

$\dfrac{x}{1}=\dfrac{y}{2}=\dfrac{z}{3}$

Direction ratios of given lines are (7,-5,1) and (1,2,3).

i.e., $a_1=7, b_1=-5, c_1=1$
$a_2=1, b_2=2, c_2=3$

Now,

$$\begin{aligned}&=a_1a_2+b_1b_2+c_1c_2\\&=7\times1+(-5)\times2+1\times3\\&=7-10+3\\&=10-10\\&=0\end{aligned}$$

$\therefore$ These two lines are perpendicular to each other.

Q14. Find the shortest distance between the $\vec{r}=\left(\hat{i}+2\hat{j}+\hat{k}\right)+\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$ and $\vec{r}=2\hat{i}+\hat{j}+\hat{k}+\mu\left(2\hat{i}+\hat{j}+2\hat{k}\right)$ **lines**

A.14. $\vec{r}=\left(\hat{i}+2\hat{j}+\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right) and -----(1)$

$\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu\left(2\hat{i}+\hat{j}+2\hat{k}\right)-----(2)$

Solution. Comparing (1) and (2) with $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ respectively.

We get,

$a_1=\hat{i}+2\hat{j}+\hat{k}, b_1=\hat{i}-\hat{j}+\hat{k}$
$a_2=2\hat{i}-\hat{j}-\hat{k}, b_2=2\hat{i}+\hat{j}+2\hat{k}$

Therefore,

$\vec{a}_2-\vec{a}_1=\hat{i}-3\hat{j}-2\hat{k}$
$\vec{b}_1\times\vec{b}_2=\left(\hat{i}-\hat{j}+\hat{k}\right)\times\left(2\hat{i}+\hat{j}+2\hat{k}\right)$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= (2-1)\hat{i} - (2-2)\hat{j} + (1+2)\hat{k}$$

$$= -3\hat{i} + 3\hat{k}$$

$$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$\left(\vec{b}_1 \times \vec{b}_2\right).\left(\vec{a}_2 - \vec{a}_1\right) = \left(-3\hat{i} + 3\hat{k}\right)\left(\hat{i} - 3\hat{j} - 2\hat{k}\right)$$

$$= -3 - 6$$

$$= -9$$

Hence, the shortest distance between the given line is given by

$$d = \left|\frac{\left(\vec{b}_1 \times \vec{b}_2\right).\left(\vec{a}_2 - \vec{a}_1\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right|$$

$$= \left|\frac{-9}{3\sqrt{2}}\right|$$

$$= \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

Q15. Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.

A.15. $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$

$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

Shortest distance between two lines is given by,

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$d = \sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2} \quad \text{--- (1)}$$

Comparing the given equation we have

$x_1 = -1, y_1 = -1, z_1 = -1$
$a_1 = 7, b_1 = -6, c_1 = 1$
$x_2 = 3, y_2 = 5, z_2 = 7$
$a_2 = 1, b_2 = -2, c_2 = 1$

Then,

$$\begin{vmatrix} (3+1) & (5+1) & (7+1) \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 4 & 6 & 8 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$= 4(-6+2) - 6(7-1) + 8(-14+6)$
$= -16 - 36 - 64$
$= -116$

And

$= \sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}$
$= \sqrt{(-6+2)^2 + (1-7)^2 + (-2+6)^2}$
$= \sqrt{16+36+16} = \sqrt{116} = 2\sqrt{29}$

Now,

$$d = \frac{-116}{2\sqrt{29}} = \frac{-58}{\sqrt{29}} = \frac{-58\sqrt{29}}{29}$$
$$d = -2\sqrt{29}$$

d= $2\sqrt{29}$ as distance can't be negative.

Q16. Find the shortest distance between the lines whose vector equations are

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + 3\hat{j} + 2\hat{k})$

and $\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu(2\hat{i} + 3\hat{j} + \hat{k})$

A.16.
$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ *and* $-----(1)$

$\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu(2\hat{i} + 3\hat{j} + \hat{k}) -----(2)$

Here, comparing (1) and (2) with $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$, we have

$a_1 = \hat{i} + 2\hat{j} + 3\hat{k}, b_1 = \hat{i} - 3\hat{j} + 2\hat{k}$

$a_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}, b_2 = 2\hat{i} + 3\hat{j} + \hat{k}$

Therefore,

$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$\vec{b}_1 \times \vec{b}_2 =$

$\hat{i}$	$\hat{j}$	$\hat{k}$
1	−3	2
2	3	1

$= \hat{i}(-3-6) - \hat{j}(1-4) + \hat{k}(3+6)$

$= -9\hat{i} + 3\hat{j} + 9\hat{k}$

$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-9)^2 + (3)^2 + (9)^2} = \sqrt{81+9+81}$

$= \sqrt{171}$

$= 3\sqrt{19}$

So,

Here, the shortest distance between the given lines is given

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$= \left| \frac{(-9\hat{i} + 3\hat{j} + 9\hat{k})(3\hat{i} + 3\hat{j} + 3\hat{k})}{3\sqrt{19}} \right|$$

$$= \left| \frac{-27+9+27}{3\sqrt{19}} \right| = \frac{9}{3\sqrt{19}}$$

$$d = \frac{3}{\sqrt{19}}$$

Q17. Find the shortest distance between the lines whose vector equations are

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} = (3-2t)\hat{k}$ and

$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} = (2s+1)\hat{k}$

A.17.

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$

$\vec{r} = \hat{i} - t\hat{i} + t\hat{j} - 2\hat{j} + 3\hat{k} - 2t\hat{k}$

$\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + t(-\hat{i} + \hat{j} - 2\hat{k})$ ------(1)

$\Rightarrow \vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

$\vec{r} = s\hat{i} + \hat{i} + 2s\hat{j} - \hat{j} - 2s\hat{k} - \hat{k}$

$\vec{r} = \hat{i} - \hat{j} - \hat{k} + s(\hat{i} + 2\hat{j} - 2\hat{k})$ ------(2)

Here,

$$a_1 = \hat{i} - 2\hat{j} + 3\hat{k}, b_1 = -\hat{i} + \hat{j} - 2\hat{k}$$

$$a_2 = \hat{i} - \hat{j} - \hat{k}, b_2 = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = \left(\hat{i} - \hat{j} - \hat{k}\right) - \left(\hat{i} - 2\hat{j} + 3\hat{k}\right)$$

$$= \hat{j} - 4\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 =$$

$\hat{i}$	$\hat{j}$	$\hat{k}$
-1	1	-2
1	2	-2

$$= (-2+4)\hat{i} - (2+2)\hat{j} + (-2-1)\hat{k}$$

$$= 2\hat{i} - 4\hat{j} - 3\hat{k}$$

$$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(2)^2 + (-4)^2 + (-3)^2}$$

$$= \sqrt{4+16+9}$$

$$= \sqrt{29}$$

$$\therefore \left(\vec{b}_1 \times \vec{b}_2\right).(\vec{a}_2 - \vec{a}_1) = \left(2\hat{i} - 4\hat{j} - 3\hat{k}\right).\left(\hat{j} - 4\hat{k}\right)$$

$$= -4 + 12$$

$$= 8$$

Hence, the shortage distance between the line is given by

$$d = \left|\frac{\left(\vec{b}_1 \times \vec{b}_2\right).(\vec{a}_2 - \vec{a}_1)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = \frac{8}{\sqrt{29}}.$$