Chapter 11: Three Dimensional Geometry

Exercise 11.1

Q1. If a line makes angles 90°, 135°, 45° with x ,y and z axes respectively, find its direction cosines.

A.1. Let the direction cosine of the line be l, m, n.

Then,

$$l = 90^\circ = \cos 90^\circ = 0$$
$$m = \cos 135 = \frac{-1}{\sqrt{2}}$$
$$n = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Q2. Find the direction cosines of a line which makes equal angles with the co-ordinate axes.

A.2. **Let the angles be α, β, r which are equal**

Let the direction cosines of the line be l, m, n.

i.e.,

$$l = \cos\alpha, m = \cos\beta, n = \cos r$$
$$\alpha = \beta = r$$
$$l^2 + m^2 + n^2 = 1$$
$$\cos^2\alpha + \cos^2\beta + \cos^2 r = 1$$
$$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$$
$$3\cos^2\alpha = 1$$
$$\cos^2\alpha = \frac{1}{3}$$
$$\cos\alpha = \frac{1}{\sqrt{3}}$$

$\therefore$ The direction cosine are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

Q3. If a line has direction ratios −18, 12, −4, then what are its direction cosines?

A.3. Direction cosine are

$$\frac{-18}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}}, \frac{+12}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}}, \frac{-4}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}}$$
$$= \frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$$
$$= \frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$$

Q4. Show that the points (2, 3, 4), (−1, −2, 1), (5, 8, 7) are collinear.

A.4. Given,

A(2,3,4), B(-1,-2,1), C(5,8,7)

Direction ratio of AB= $(-1-2),(-2-3),(1-4)$
$=(3,-5,-3)$

Where, $a_1=3$, $b_1=-5$, $c_1=-3$

Direction ratio of BC= $(5-(-1)),(8-(-2)),(7-1)$
$=(6,10,6)$

Where, $a_2=6$, $b_2=10$, $c_2=6$

Now,

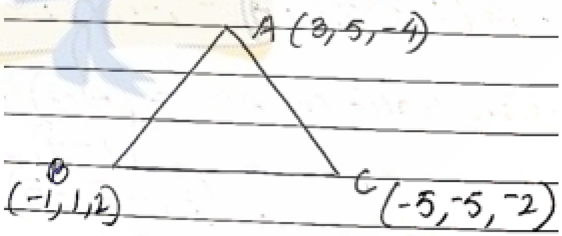
$$\frac{a_2}{a_1}=\frac{6}{-3}=-2$$
$$\frac{b_2}{b_1}=\frac{10}{-5}=-2$$
$$\frac{c_2}{c_1}=\frac{6}{-3}=-2$$

Here, direction ratio of two-line segments are proportional.

So, A, B, C are collinear.

Q5. Find the direction cosines of the sides of the triangle whose vertices are (3, 5, − 4), (− 1, 1, 2) and (− 5, − 5, − 2)

A.5. The vertices of $\triangle$ABC are A(3,5,-4), B(-1,1,2) and C(-5,-5,-2)



Direction ratio of side AB $=(-1-3)(1-5)(2-(-4))$
$=(-4,-4,6)$

Direction cosine of AB,

$$=\frac{-4}{\sqrt{(-4)^2+(-4)^2+6^2}},\frac{-4}{\sqrt{(-4)^2+(-4)^2+6^2}},\frac{6}{\sqrt{(-4)^2+(-4)^2+6^2}}$$

$$=\frac{-4}{\sqrt{16+16+36}},\frac{-4}{\sqrt{16+16+36}},\frac{6}{\sqrt{16+16+36}}$$

$$=\frac{-4}{\sqrt{68}},\frac{-4}{\sqrt{68}},\frac{6}{\sqrt{68}}$$

$$=\frac{-4}{2\sqrt{17}},\frac{-4}{2\sqrt{17}},\frac{6}{2\sqrt{17}}$$

$$=\frac{-2}{\sqrt{17}},\frac{-2}{\sqrt{17}},\frac{3}{\sqrt{17}}$$

Direction ratios of BC= $(-5-(-1)),(-5-1),(-2-2)$
$=(-4,-6,-4)$

Direction cosine of BC

$$=\frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}},\frac{-6}{\sqrt{(-4)^2+(-6)^2+(-4)^2}},\frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}$$

$$=\frac{-4}{2\sqrt{17}},\frac{-6}{2\sqrt{17}},\frac{-4}{2\sqrt{17}}$$

$$=\frac{-2}{\sqrt{17}},\frac{-3}{\sqrt{17}},\frac{-2}{\sqrt{17}}$$

Direction of CA= $(-5-3)(-5-5)(-2-(-4))$
$=(-8,-10,2)$

Direction cosine of CA

$$=\frac{-8}{\sqrt{(-8)^2+(-10)^2+2^2}},\frac{-10}{\sqrt{(-8)^2+(-10)^2+2^2}},\frac{2}{\sqrt{(-8)^2+(-10)^2+2^2}}$$

$$=\frac{-8}{\sqrt{168}},\frac{-10}{\sqrt{168}},\frac{2}{\sqrt{168}}$$

$$=\frac{-8}{2\sqrt{42}},\frac{-10}{2\sqrt{42}},\frac{2}{2\sqrt{42}}$$

$$=\frac{-4}{\sqrt{42}},\frac{-5}{\sqrt{42}},\frac{1}{\sqrt{42}}$$