Exercise 10.2

Q1 Compute the magnitude of the following vectors:

$\vec{a} = \hat{i} + \hat{j} + \hat{k}; \quad \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}; \quad \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$

A.1. $\vec{a} = \hat{i} + \hat{j} + \hat{k}; \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}; \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$

Magnitude of $|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

Magnitude of $\left|\vec{b}\right| = \sqrt{2^2 + (-7)^2 + (-3)^2} = \sqrt{4+49+9} = \sqrt{62}$

Magnitude of $\left|\vec{c}\right| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = \sqrt{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = \sqrt{\frac{3}{3}} = 1$

Q2. Write two different vectors having same magnitude.

A.2. Two different vectors having same magnitude: -

(i) $2\hat{i} + \hat{j} + 3\hat{k}$

(ii) $\hat{i} + 3\hat{j} + 2\hat{k}$

Q3. Write two different vectors having same direction.

A.3. Two different vectors having same directions: -

(i) $\hat{i} + \hat{j} + \hat{k}$

(ii) $2\hat{i} + 2\hat{j} + 2\hat{k}$

Q4. Find the values of *x* and *y* so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ **are equal.**

A.4. Note that two vector are equal only if their corresponding components are equal.

Thus, the given vectors $\vec{a}$ and $\vec{b}$ will be equal if and only if $x = 2 \,\&\, y = 3$

Q5. Find the scalar and vector components of the vector with initial point (2, 1) and terminal point (–5, 7).

A.5. Let the vector with initial point P (2,1) and terminal point Q. (-5,7) can be shown as,

$$\overrightarrow{PQ} = (-5,-2)\hat{i} + (7,-1)\hat{j}$$

$$\overrightarrow{PQ} = -7\hat{i} + 6\hat{j}$$

The scalar components are -7 and 6.

The vector components are -7i and 6j.

Q6. Find the sum of the vectors: $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=-2\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c}=\hat{i}-6\hat{j}-7\hat{k}$

A.6. The given vectors are

$$\vec{a}=\hat{i}-2\hat{j}+\hat{k}$$

$$\vec{b}=-2\hat{i}+4\hat{j}+5\hat{k}$$

$$\vec{c}=\hat{i}-6\hat{j}-7\hat{k}$$

The sum of the vector is

$$\vec{a}+\vec{b}+\vec{c}=\left(a_1+a_2+a_3\right)\hat{i}+\left(b_1+b_2+b_3\right)\hat{j}+\left(c_1+c_2+c_3\right)\hat{k}$$

$$=(1-2+1)\hat{i}+(-2+4-6)\hat{j}+(1+5-7)\hat{k}$$

$$=0.\hat{i}+(-4)\hat{j}+(-1)\hat{k}$$

$$=-4\hat{j}-\hat{k}$$

Q7. Find the unit vector in the direction of the vector $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$

A.7. The given vectors is $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$

So, $\left|\vec{a}\right|=\sqrt{1^2+1^2+2^2}=\sqrt{1+1+4}=\sqrt{6}$

The required unit vector is given by= $\hat{a}=\frac{1}{\left|\vec{a}\right|}.\vec{a}$

$$\hat{a}=\frac{1\left(\hat{i}+\hat{j}+2\hat{k}\right)}{\sqrt{6}}=\frac{\hat{i}}{\sqrt{6}}+\frac{\hat{j}}{\sqrt{6}}+\frac{2\hat{k}}{\sqrt{6}}$$

Q8. Find the unit vector in the direction of the vector $\overrightarrow{PQ}$ **where P and Q. are the points (1, 2, 3) and (4, 5, 6) respectively.**

A.8. Given, $\mathbf{P}\left(1,2,3\right)$ & $\mathbf{Q}\left(4,5,6\right)$

So,

$$\overrightarrow{PQ}=(4-1)\hat{i}+(5-2)\hat{j}+(6-3)\hat{k}$$

$$=3\hat{i}+3\hat{j}+3\hat{k}$$

$$\left|\overrightarrow{PQ}\right|=\sqrt{3^2+3^2+3^2}=\sqrt{9+9+9}=\sqrt{27}=3\sqrt{3}$$

The unit vector in the3 direction of $\overrightarrow{PQ}$ is

$$=\frac{3\hat{i}}{3\sqrt{3}},\frac{3\hat{j}}{3\sqrt{3}},\frac{3\hat{k}}{3\sqrt{3}}$$

$$=\frac{\hat{i}}{\sqrt{3}}+\frac{\hat{j}}{\sqrt{3}}+\frac{\hat{k}}{\sqrt{3}}$$

Q9. For given vectors $a=2\hat{i}-\hat{j}+2\hat{k}$ and $b=-\hat{i}+\hat{j}-\hat{k}$**, find the unit vector in the direction of** $\vec{a}+\vec{b}$

A9. Given, $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}\ \&\ \vec{b}=-\hat{i}+\hat{j}-\hat{k}$

The sum of given vectors is given by

$$\vec{a}+\vec{b}\left(=\vec{c}, say\right)=(2-1)\hat{i}+(-1+1)\hat{j}+(2-1)\hat{k}$$

$$=\hat{i}+0.\hat{j}+\hat{k}$$

$$=\hat{i}+\hat{k}$$

$$\left|\vec{c}\right|=\sqrt{1^2+1^2}=\sqrt{2}$$

The unit vector in the direction of $\left(\vec{a}+\vec{b}\right)=\vec{c}$ is given by

$$\hat{c}=\frac{1}{\left|\vec{c}\right|}=\frac{\hat{i}+\hat{k}}{\sqrt{2}}=\frac{\hat{i}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}$$

Q10. Find the vector in the direction of vector $5\hat{i}-\hat{j}+2\hat{k}$ **which has magnitude 8 units.**

A.10. Let, $\vec{a}=5\hat{i}-\hat{j}+2\hat{k}$

$$\left|\vec{a}\right|=\sqrt{5^2+(-1)^2+2^2}=\sqrt{25+1+4}=\sqrt{30}$$

The unit vector in the direction of the given vector $\vec{a}$ is

$$\hat{a} = \frac{1}{|\vec{a}|}.\vec{a} = \frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}$$

Therefore, the vector having magnitude equal to 8 and in direction of $\vec{a}$ is

$$8\hat{a} = 8\left(\frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}\right) = \frac{40\hat{i}}{\sqrt{30}} - \frac{8\hat{j}}{\sqrt{30}} + \frac{16\hat{k}}{\sqrt{30}}$$

Q11. Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$ **are collinear.**

A.11. Let, $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k} \;\&\; \vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$

It is seen that

$$\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$$

$$= -2\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right)$$

$$= -2\vec{a}$$

$$\therefore \vec{b} = \lambda\vec{a}$$

Where, $\lambda = -2$

Therefore, we can say that the given vector are collinear.

Q12. Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.

A.12. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

The modulus is given by; $|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$

Let, l, m and n are direction cosines of the given vector, then

$$l = \frac{x}{|\vec{a}|} = \frac{1}{\sqrt{14}}; m = \frac{y}{|\vec{a}|} = \frac{2}{\sqrt{14}}; n = \frac{z}{|\vec{a}|} = \frac{3}{\sqrt{14}}$$

$\therefore$ Thus, the direction cosines of $\vec{a}$ are

$$\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$$

Q13. Find the direction cosines of the vector joining the points A (1, 2, –3) and B (–1, –2, 1) directed from A to B.

A.13. Given, A(1,2,-3) and (-1,-2,1)

Now,

$$\left|\overrightarrow{AB}\right| = (-1-1)\hat{i} + (-2-2)\hat{j} + (1-(-3))\hat{k}$$

$$= -2\hat{i} - 4\hat{j} + 4\hat{k}$$

Then,

$$\left|\overrightarrow{AB}\right| = \sqrt{(-2)^2 + (-4)^2 + (4)^2}$$

$$\sqrt{4+16+16} = \sqrt{36} = 6$$

Let, l, m, n be direction cosine,

$$l = \frac{x}{\left|\overrightarrow{AB}\right|} = \frac{-2}{6} = \frac{-1}{3}; m = \frac{y}{\left|\overrightarrow{AB}\right|} = \frac{-4}{6} = \frac{-2}{3}; n = \frac{z}{\left|\overrightarrow{AB}\right|} = \frac{4}{6} = \frac{2}{3}$$

Therefore, direction cosine of $\overrightarrow{AB}$ are $\left(\frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}\right)$

Q14. Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY and OZ.

A.14. Here,

Let, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

Then,

$$\left|\vec{a}\right| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

Here, the directions cosines of $\vec{a}$ are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Now, let $\alpha, \beta \,\&\, \chi$ be the angle formed by $\vec{a}$ with the positive directions of x, y & z axis.

So, we have

$$\cos\alpha = \frac{1}{\sqrt{3}}, \cos\beta = \frac{1}{\sqrt{3}}, \cos\chi = \frac{1}{\sqrt{3}}$$

Therefore, the given vector is equally inclined to axis OX, OY and OZ.

Q15. Find the position vector of a point R which divides the line joining two points P and Q. whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and -$\hat{i}$ + $\hat{j} + \hat{k}$ respectively, in the ratio 2 : 1

(i) internally

(ii) externally.

A.15. (i) The position vector of point R dividing the join of P and Q. internally in the ratio 2:1 is,

$$= \frac{\left(-2\hat{i}+\hat{i}\right)+\left(2\hat{j}+2\hat{j}\right)+\left(2\hat{k}+\hat{k}\right)}{3} = \frac{-\hat{i}+4\hat{j}+\hat{k}}{3}$$

$$= -\frac{1}{3}\hat{i}+\frac{4}{3}\hat{j}+\frac{1}{3}\hat{k}$$

(ii) The position vector of the point k dividing the join of P and Q. externally in the ratio 2:1

A15. (ii)

$$\overrightarrow{OR} = \frac{2\left(-\hat{i}+\hat{j}+\hat{k}\right)-1\left(\hat{i}+2\hat{j}-\hat{k}\right)}{2-1}$$

$$= -2\hat{i}+2\hat{j}+2\hat{k}-\hat{i}-2\hat{j}+\hat{k}$$

$$= -3\hat{i}+\hat{k}$$

Q16. Find the position vector of the mid-point of the vector joining the points P (2, 3, 4) and Q. (4, 1, – 2).

A.16.The Position vector of mid-point R of the vector joining point P (2,3,4) and Q (4,1,-2) is given by;

$$\overrightarrow{OR} = \frac{\left(2\hat{i}+3\hat{j}+4\hat{k}\right)+\left(4\hat{i}+\hat{j}-2\hat{k}\right)}{2}$$

$$= \frac{(2+4)\hat{i}+(3+1)\hat{j}+(4-2)\hat{k}}{2}$$

$$= \frac{6\hat{i}+4\hat{j}+2\hat{k}}{2} = \frac{6\hat{i}}{2}+\frac{4\hat{j}}{2}+\frac{2\hat{k}}{2}$$

$$= 3\hat{i}+2\hat{j}+\hat{k}$$

Q17. Show that the points A, B and C with position vectors $\vec{a} = 3\hat{i}-4\hat{j}-4\hat{k}, \vec{b} = 2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c} = \hat{i}-3\hat{j}-5\hat{k}$ **respectively form the vertices of a right angled triangle.**

A.17. We have,

$\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$

$\overrightarrow{AB} = (2-3)\hat{i} + (-1-(-4))\hat{j} + (1-(-4))\hat{k}$

$= -\hat{i} + 3\hat{j} + 5\hat{k}$

$\overrightarrow{BC} = (1-2)\hat{i} + (-3-(-1))\hat{j} + (-5-1)\hat{k}$

$= -\hat{i} - 2\hat{j} - 6\hat{k}$

$\overrightarrow{CA} = (1-3)\hat{i} + (-3-(-4))\hat{j} + (-5-(-4))\hat{k}$

$= -2\hat{i} + \hat{j} - \hat{k}$

Now,

$\left|\overrightarrow{AB}\right| = \sqrt{(-1)^2 + (3)^2 + (5)^2} = \sqrt{1+9+25} = \sqrt{35}$

$\left|\overrightarrow{BC}\right| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{1+4+36} = \sqrt{41}$

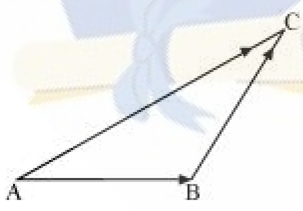
$\left|\overrightarrow{CA}\right| = \sqrt{(-2)^2 + (1)^2 + (-1)^2} = \sqrt{4+1+1} = \sqrt{6}$

Hence,

$\left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{CA}\right|^2 = 35 + 6 = 41 = \left|\overrightarrow{BC}\right|^2$

Hence, given points from the vertices of a right angled triangle.

Q18. In triangle ABC (Fig. below), which of the following is not true:



A. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

B. $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

C. $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$

D. $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

A18. (A) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$

By triangle law of addition in given triangle, we get:

$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}-----(1)$

$\overrightarrow{AB}+\overrightarrow{BC}=-\overrightarrow{CA}$

$\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}-----(2)$

So, (A) is true.

(B) $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC}=0$

$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

$\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC}=0$

So, (B) is true.

(C) $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{CA}=0$

$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{CA}-----(3)$

$From, (1) \,\&\, (3),$

$\overrightarrow{AC}=\overrightarrow{CA}$

$\overrightarrow{AC}=-\overrightarrow{AC}$

$\overrightarrow{AC}+\overrightarrow{AC}=0$

$2\overrightarrow{AC}=0$

$\therefore$ The eQ.uation in alternative C $\overrightarrow{AC}=\vec{0}$, which is not true, is incorrect.

(D) $\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA}=\vec{0}$

$From, eqn (2) we have$

$\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA}=0$

The, equation given is alternative is D is true.

$\therefore$ The correct answer is C.

Q.19. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect:

(A) $\vec{b}=\lambda\vec{a}$ **for some scalar λ**

(B) $\vec{a} = \pm\vec{b}$

(C) The respective components of $\vec{a}$ and $\vec{b}$ are proportional.

(D) Both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes.

A.19. We know,

If $\vec{a}$ and $\vec{b}$ are two collinear vector, they are parallel.

So,

$$\vec{b} = \lambda\vec{a}$$

$$If, \lambda = \pm 1, then, \vec{a} = \pm\vec{b}$$

$$If, \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, then$$

$$\vec{b} = \lambda\vec{a}$$

$$b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \lambda\left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$$

$$= (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$$

$$b_1 = \lambda a_1, b_2 = \lambda a_2, b_3 = \lambda a_3$$

$$\frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda$$

Hence, the respective component are proportional but, vector $\vec{a}$ and $\vec{b}$ can have different direction.

Thus, the statement given in D is incorrect.

The correct answer is D.