Q1. Show that the function f: R* → R* defined by f(x) = 1/x is one-one and onto, where: R* is the set of all non-zero real numbers. Is the result true, if the domain: R* is replaced by N with co-domain being same as: R*?

A.1. The fx^n is $f(x)=\frac{1}{x}$, which is a $f: R_* \to R_*$ and R_* is set of all non-zero real numbers

For, $x_1, x_2 \in R_*$, $f(x_1)=f(x_2)$

$$\Rightarrow \frac{1}{x_1}=\frac{1}{x_2}$$

$\Rightarrow x_1 = x_2$ So, f is one-one

For, $y \in R_*$, $x=\frac{1}{f(x)}=\frac{1}{y}$ such that

So, $f(x)=y$

So, every element in the co-domain has a pre-image in f

So, f is onto

If $f: N \to R_*$ such that $f(x)=\frac{1}{x}$

For, $x_1, x_2 \in N$, $f(x_1)=f(x_2)$

$$\Rightarrow \frac{1}{x_1}=\frac{1}{x_2}$$

$\Rightarrow x_1 = x_2$ So, f is one-one

For, $y \in R_*$ and $f(x)=y$ we have $x=\frac{1}{y} \notin N$

Eg., $3 \in R_*$ so $x=\frac{1}{3} \notin N$

So, f is not onto

Q2. Check the injectivity and surjectivity of the following functions:

(i) f: N → N given by f(x) = x^2

(ii) f: Z → Z given by f(x) = x^2

(iii) f: R → R given by f(x) = x^2

(iv) f: N → N given by f(x) = x^3

(v) f: Z → Z given by f(x) = x^3

A.2. i) $f: N \to N$ given by $f(x) = x^2$

For, $x_1, x_2 \in N$, $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2 \notin N$

So, f is one-one/ injective

For $x \in N$, i.e., $x = 1, 2, 3....$

Range of $f(x) = \{1^2, 2^2, 3^2...\} = \{1, 4, 9...\} \neq N$

i.e., co-domain of N

So, f is not onto/ subjective

ii) $f: Z \to Z$ given by $f(x) = x^2$

For, $x_1, x_2 \in Z$, $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2 \notin Z$

i.e., $x_1 = x_2$ and $x_1 = -x_2$

So, f is not one-one/ injective

For $x \in Z$, $x = 0, \pm 1, \pm 2, \pm 3....$

Range of $f(x) = \{0^2, (\pm 1)^2, (\pm 2)^2, (\pm 3)^2...\}$

$\{0, 1, 4, 9....\} \neq$ co-domain Z

So, f is not onto/ subjective

iii) $f: R \to R$ given by $f(x) = x^2$

For, $x_1, x_2 \in R$, $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2$

So, f is not injective

For $x \in R$

Range of $f(x) = \{x^2, x \in R\}$ gives a set of all positive real numbers

Hence, range of $f(x \neq)$ co-domain of R

So, f is not subjective

iv) $f : N \to N$ given by $f(x) = x^3$

For, $x_1, x_2 \in N$, $f(x_1) = f(x_2)$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2 \in N$

So, f is injective

For $x \in N$,

Range of $f(x) = \{1^3, 2^3, 3^3 ....\}$

$\{1, 8, 27 ....\} \neq$ co-domain of N

So, f is not subjective

v) $f : R \to R$ given by $f(x) = x^3$

For, $x_1, x_2 \in N$, $f(x_1) = f(x_2)$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2 \in N$

So, f is injective

For $x \in R$,

Range of $f(x) = \{1^3, 2^3, 3^3 ....\}$

$\{1, 8, 27 ....\} \neq$ co-domain of R

So, f is not subjective

Q3. Prove that the Greatest Integer Function f: R → R given by f(x) = [x], is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

A.3. The fx^n $f:R\to R$ is given by $f(x)=[x]$

Let $x_1=1.5$ and $x_2=1.2\in R$ Then,

$f(x_1)=f(1.5)=[1.5]=1$

$f(x_2)=f(1.2)=[1.2]=1$

So, $f(x_1)=f(x_2)$ but $x_1\neq x_2$

i.e., $f(1.5)=f(1.2)$ but $1.5\neq 1.2$

So, f is not one-one

The range of $f(x)$ is a set of all integers, Z which is not a co-domain of R

$\therefore f$ is not onto

Q4. Show that the Modulus Function f : R → R, given by f(x) = |x| is neither one-one nor onto, where is |x| if x is positive or 0 and |-x| is -x if x is negative.

A.4. The fx^n $f:R\to R$ is given by $f(x)=|x|$

$$\Rightarrow f(x)=\begin{pmatrix} x\ ,if & x\geq 0 \\ -x\ ,if & x<0 \end{pmatrix}$$

For $x_1=-1$ and $x_2=1$

$f(x_1)=f(-1)=|-1|=1$

$f(x_2)=f(1)=|1|=1$

So, $f(x_1)=f(x_2)$ but $x_1\neq x_2$

i.e., f is not one-one

For $x=-1\in R$

$f(x)=|x|$

i.e., $f(-1)=|-1|=1$

So, range of $f(x)$ is always a positive real number and is not equal to the co-domain R

i.e., f is not onto

Q5. Show that the Signum Function *f*: R → R , given by

$$f(x)=\begin{cases}1 & \text{if} \quad x>0\\ 0 & \text{if} \quad x=0\\ -1 & \text{if} \quad x<0\end{cases}$$ is neither one-one nor onto.

A.5. The fx^n $f:R\to R$ is given by $f(x)=\begin{pmatrix}1 & if & x>0\\ 0 & if & x=0\\ -1 & if & x<0\end{pmatrix}$

For $x_1=1, x_2=2, \in R$

$f(x_1)=f(1)=1$

$f(x_2)=f(2)=1$ but $1\neq 2$

So, f is not one-one

And the range of $f(x)=\{1,0,-1\}$ hence it is not equal to the co-domain R

So, f is not onto

Q6. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one.

A.6. Given, $f:A\to B$ and $f=\{(1,4),(2,5),(3,6)\}$

$\therefore f(1)=4$
$f(2)=5$
$f(3)=6$

i.e., the image elements of A under the given f_X^n f are unique

So, f is one-one

Q7. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(i) f: R → R defined by f(x) = 3 − 4x

(ii) f: R → R defined by f(x) = 1 + x²

A.7. i) $f:R\to R$ defined as $f(x)=3-4x$

For $x_1,x_2\in R$ such that $f(x_1)=f(x_2)$

$$\Rightarrow 3-4x_1 = 3-4x_2$$
$$\Rightarrow 4x_1 = 4x_2$$
$$\Rightarrow x_1 = x_2$$

So, f is one-one

For $y \in R$, there exist

$$f\left(\frac{.3-y}{4}\right) = 3-4\left(\frac{3-y}{4}\right) = 3-3+y = y$$

Hence, f is onto

$\therefore f$ is bijective

ii) Given, $f : R \to R$ defined as $f(x) = 1+x^2$

For $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$$\Rightarrow 1+x_1^2 = 1+x_2^2$$
$$\Rightarrow x_1^2 = x_2^2$$
$$\Rightarrow x_1 = \pm x_2$$
$$\Rightarrow x_1 = x_2 \text{ or } x_1 = -x_2$$

$\therefore f$ is not one-one

The range of $f(x)$ is always a positive real number which is not equal to co-domain R

So, f is not onto

Q8. Let A and B be sets. Show that f: A × B → B × A such that (a, b) = (b, a) is bijective function.

A.8. Given, $f : A \times B \to B \times A$ defined as $f(a,b) = (b,a)$

Let $(a_1, b_1), (a_2, b_2) \in A \times B$ such that

$$f(a_1, b_1) = f(a_2, b_2)$$
$$\Rightarrow (b_1, a_1) = (b_2, a_2)$$

So, $b_1 = b_2$ and $a_1 = a_2$

$$\Rightarrow (a_1, b_1) = (a_2, b_2)$$

$\therefore f$ is one-one

For $(a,b) \in B \times A$

There exist $(a,b) \in A \times B$ such that $f(a,b) = (b,a)$

$\therefore f$ is onto

Hence, f is bijective

Q9. Let N → N be defined by f(n) for all

$$f(n) = \begin{cases} \frac{n+1}{2} \text{ if n is odd} \\ \quad \text{for all } n \in N \\ \frac{n}{2} \text{ if n is even} \end{cases}$$

State whether the function f is bijective. Justify your answer.

A.9. Given, $f: N \to N$ defined $f(x) = \begin{pmatrix} \frac{x+1}{2}, if & x \text{ is odd} \\ \frac{x}{2}, if & x \text{ is even} \end{pmatrix} \cup x \in N$

Let $x_1 = 1$ and $x_2 = 2 \in N$,

$f(x_1) = f(x_2) \Rightarrow f(1) = f(2)$

$\Rightarrow \frac{1+1}{2} = \frac{2}{2}$

$\Rightarrow 1 = 1$ but $1 \neq 2$

So, f is not one-one

For $x =$ odd and $x \in N$, say $x = 2C + 1$ where $C \in N$

There exist $(4C+1) \in N$ such that

$(4C+1) = \frac{4C+1+1}{2} = 2C+1. \in N$

And for $x =$ even $\in N$, say $x = 2C$ where $C \in N$

There exist $(4C) \in N$ such that

$f(4C) = \frac{4C}{2} = 2C. \in N$

So, f is onto

But, f is not bijective

Q10. Let A = R – {3} and B = R – {1}. Consider the function f : A → B defined by f(x) = (x-2/x-3) Is f one-one and onto? Justify your answer.

A.10. Given, $f: A \to B$ defined by $f(x)=\left(\dfrac{x-2}{x-3}\right)$

Let $x_1, x_2 \in A = R-\{3\}$ such that

$$f(x_1)=f(x_2)$$

$$\Rightarrow \frac{x_1-2}{x_1-3}=\frac{x_2-2}{x_2-3}$$

$$\Rightarrow (x_1-2)(x_2-3)=(x_2-2)(x_1-3)$$

$$\Rightarrow x_1x_2-3x_1-2x_2+6=x_2x_1-3x_2-2x_1+6$$

$$\Rightarrow 2x_1-3x_1=2x_2-3x_2$$

$$\Rightarrow -x_1=-x_2$$

$$\Rightarrow x_1=x$$

So, f is one-one

For $y \in B = R-\{1\}$ there exist $f(x)=y$ such that

$$\frac{x-2}{x-3}=y$$

$$\Rightarrow x-2=xy-3y$$

$$\Rightarrow x-xy=2-3y$$

$$\Rightarrow x(1-y)=2-3y$$

$$\Rightarrow x=\frac{2-3y}{1-y} \text{ where } y\neq 1.\in A$$

Thus, $f\left(\dfrac{2-3y}{1-y}\right)=\dfrac{\left(\dfrac{2-3y}{1-y}\right)-2}{\left(\dfrac{2-3y}{1-y}\right)-3}=\dfrac{2-3y-2+2y}{2-3y-3+3y}$

$$\Rightarrow \frac{-y}{-1}=y$$

$\therefore f$ is onto

Q11. Let f: R → R be defined as f(x) = x⁴. Choose the correct answer.

(A) f is one-one onto

(B) f is many-one onto

(C) f is one-one but not onto

(D) f is neither one-one nor onto

A.11. Given, $f: R \rightarrow R$ defined by $f(x) = x^4$

For $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^4 = x_2^4$

$\Rightarrow x_1 = \pm x_2$

$\Rightarrow x_1 = x_2$ or $x_1 = -x_2$

So, f is not one-one

The range of $f(x)$ is a set of all positive real numbers which is not equal to co-domain R

So, f in not onto

$\therefore$ Option (D) is correct

Q12. Let f: R → R be defined as f(x) = 3x. Choose the correct answer.

(A) f is one-one onto

(B) f is many-one onto

(C) f is one-one but not onto

(D) f is neither one-one nor onto

A.12. Given, $f: R \rightarrow R$ defined as $f(x) = 3x$

For $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow 3x_1 = 3x_2$

$\Rightarrow x_1 = x_2$

So, f is one-one

And for $y \in R$, there exist $\frac{y}{3} \in R$ such that

$f\left(\frac{y}{3}\right) = \frac{3 \times y}{3} = y$

$\therefore f$ is onto

Hence, option (A) is correct