Chapter 1: Relations and Functions

Ex 1.1

Q1. Determine whether each of the following relations are reflexive, symmetric and transitive:

(i)Relation R in the set A = {1, 2, 3…13, 14} defined as

R = {(x, y): 3x − y = 0}

(ii) Relation R in the set N of natural numbers defined as

R = {(x, y): y = x + 5 and x < 4}

(iii) Relation R in the set A = {1, 2, 3, 4, 5, 6} as

R = {(x, y): y is divisible by x}

(iv) Relation R in the set Z of all integers defined as

R = {(x, y): x − y is as integer}

(v) Relation R in the set A of human beings in a town at a particular time given by

(a) R = {(x, y): x and y work at the same place}

(b) R = {(x, y): x and y live in the same locality}

(c) R = {(x, y): x is exactly 7 cm taller than y}

(d) R = {(x, y): x is wife of y}

(e) R = {(x, y): x is father of y}

A.1. (i) We have, $R=\{(x,y):3x-y=0\}$ a relation in set A= $\{1,2,3..........14\}$

For $x\in A, y=3x$ or $y\neq x$ i.e.,

(x,x) does not exist in R

$\therefore$ R is not reflexive.

For $(x,y)\in R, y=3x$

Then $(y,x)\, x\neq 3y$

So $(y, x) \notin R$

$\therefore$ R is not symmetric

For $(x, y) \in R$ and $(y, z) \in R$. We have

$y = 3x$ and $z = 3y$

Then $z = 3(3x) = 9x$

i.e., $(x, z) \notin R$

$\therefore$ R is not Transitive

(ii) We have,

R= $\{(x, y) : y = x + 5 \,\&\, x < 4\}$ is a relation in N

= $\{(1, 1+5), (2, 2+5), (3, 3+5)\}$

=$\{(1, 6), (2, 7), (3, 8)\}$

Clearly, R is not reflexive as $(x, x) \notin R$ and $x < 4 \,\&\, x \in N$

Also, R is not symmetric as $(1, 6) \in R$ but $(6, 1) \notin R$

And for $(x, y) \in R\,(y, z) \notin R$. Hence, R is not Transitive.

(iii) R=$\{(x, y);\, y$ is divisible by $x\}$ is a relation in set

A=$\{1, 2, 3, 4, 5, 6\}$

So, R=$\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (5,5), (6,6)\}$

Hence, R is reflexive because $(1,1), (2,2), (3,3), (4,4), (5,5), (6,6) \in R$ i.e., $(x, x) \in R$

R is not symmetric as $(1, 2) \in R$ but $(2,1) \notin R$

And for $x, y, z \in A$ and $(x, y) \in R \,\&\, (y, z) \in R$

$\frac{y}{z} = n$ and $\frac{z}{y} = m$ where $n \,\&\, m \in N$

Then, $z = my = m(nx) = mn.x$

$\Rightarrow \dfrac{z}{x} = m.n, m, mn \in N$

Hence, $(x, z) \in R$

$\therefore$ R is transitive

(iv) $R = \{(x, y) : x - y$ is an integer $\}$ is a relation in set Z

For $x \in Z$

$x - x = 0$ is an integer

So, $(x, x) \in R$ i.e., R is reflexive

For $x, y \in Z$ and $(x, y) \in R$

$x - y$ is an integer

$-(y - x)$ is an integer

$(y - x)$ is an integer

So, $(y, x) \in R$ i.e., R is symmetric

For $x, y, z \in R$ and $(x, y) \in R$ & $(y, z) \in R$ We have,

$x - y$ is an integer

$y - z$ is an integer

So, $(x - y) + (y - z)$ is also an integer

$x - z$ is an integer

So, $(x, z) \in R$ i.e., R is transitive.

(v) (a) $R = \{(x, y) : x$ and y work at same place $\}$ in set A of human being.

For $x \in A$ we get

x & x work at same place

$(x, x) \in R$ So, R is reflexive.

For $x, y \in A$ and $(x, y) \in R$ We get,

$x \& y$ work at same place

$y \& x$ work at same place

$(y, x) \in R$. So, R is symmetric.

For $x, y, z \in A$ and (x, y) and $(y, z) \in R$. We have,

$x \& y$ work at same place

$y \& z$ work at same place

$x \& z$ work at same place

i.e., $(x, z) \in R$. So, R is Transitive.

(b) $R = \{(x, y) : x \& y$ live in same locality $\}$

For $x \in A$, we get,

$x \& x$ live in same locality

$(x, x) \in R$ So, R is reflexive.

For $x, y \in A$ and $(x, y) \in R$ we get,

$x \& y$ live in same locality

$y \& x$ live in same locality

i.e., $(y, x) \in R$. So, R is symmetric.

For $x, y, z \in A$ and $(x, y) \& (y, z) \in R$. Then

x, y live in same locality

y, z live in same locality

x, z live in same locality

So, $(x, z) \in R$ i.e., R is transitive.

(c) $R = \{(x, y) : x$ is exactly 7 cm taller than $y \}$

For $x \in A$,

Height $(x) \neq 7 +$ height of (x)

So, $(x, x) \notin R$. i.e., R is not reflexive.

For, $x, y \in A$ and $(x, y) \in R$ we have,

$\text{Height}(x) = 7 + \text{height}(y)$

But $\text{height}(y) \neq 7 + \text{height}(x)$

So, $(y, x) \in R$ i.e., R is not symmetric.

For $x, y, z \in A$ and $(x, y) \in R$ and $(y, z) \in R$ we have,

$\text{Height}(x) = 7 + \text{height}(y)$

And $\text{height}(y) = 7 + \text{height}(z)$

So, $\text{height}(x) = 7 + 7 + \text{height}(z) = 14 + \text{height}(z)$

i.e., $(x, z) \notin R$

So, R is not transitive.

(d) $R = \{(x, y) : x \text{ is wife of } y\}$

For $x \in A$,

X is not wife of x.

So, $(x, x) \notin R$ i.e., R is not reflexive.

For $x, y \in A$ and $(x, y) \in R$,

X is wife of y but y is not wife of x

So, $(y, x) \notin R$. i.e., R is not symmetric.

For $x, y, z \in A$ and (x, y) and $(y, z) \in R$

X is wife of y

Y is wife of z

But y can never be husband & wife simultaneously

So, R is not transitive.

(e) $R = \{(x, y) : x \text{ is father of } y\}$

For $x \in A$,

X is not a father of x

So, $(x, x) \notin R$ i.e., R is not reflexive.

For $x, y \in A$ and $(x, y) \in R$

X is father of y

Y is father of z

But x is not father of z

So, $(x, z) \notin R$ i.e., R is not transitive.

Q2. Show that the relation R in the set R of real numbers, defined as

R = {(a, b): a ≤ b²} is neither reflexive nor symmetric nor transitive.

A.2. We have,

$R=\{(a, b): a \le b^2\}$ is a relation in R.

For $a \in R$ then is $b = a, a \le a^2$ is not true for all real number less than 1.

Hence, R is not reflexive.

Let $(a, b) \in R$ and a=1 and b=2

Then, $a \le b^2 = 1 \le 2^2 = 1 \le 4$ so, $(1, 2) \in R$

But $(b, a) = (2, 1)$

i.e., $2 \le 1^2 = 2 \le 1$ is not true

so, $(2, 1) \notin R$

hence, R is not symmetric.

For, $(a, b) = (10, 4) \& (b, c) = (4, 2) \in R$

We have, $a = 10 \le 4^2 = b^2 => 10 \le 16$ is true

So, $(10, 4) \in R$

And $4 \le 2^2 => 4 \le 4$ So, $(4, 2) \in R$

But $10 \le 2^2 => 10 \le 4$ is not true.

So, $(10, 2) \notin R$

Hence, R is not transitive.

Q3. Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as

R = {(a, b): b = a + 1} is reflexive, symmetric or transitive.

A.3. We have,

R=$\{(a,b):b=a+1\}$ is a relation in set $\{1,2,3,4,5,6\}$

So, R=$\{(1,2),(2,3),(3,4),(4,5),(5,6),(6,7)\}$

As, $(1,1)\notin R$, R is not reflexive

As, $(1,2)\in R$ but $(2,1)\notin R$, R is not symmetric

And as $(1,2)$ & $(2,3)\in R$ but $(1,3)\notin R$

Hence, R is not transitive.

Q4. Show that the relation R in R defined as R = {(a, b): a ≤ b}, is reflexive and transitive but not symmetric.

A.4. We have, R=$\{(a,b):a\le b\}$ is a relation in R.

For, $a\in R$,

$a\le b$ but $b\le a$ is not possible i.e., $(b,a)\notin R$

Hence, R is not symmetric.

For $(a,b)\in R$ & $(b,c)\in R$ and $a,b,c\in R$

$a\le b$ and $b\le c$

So, $a\le c$

i.e., $(a,c)\in R$

$\therefore$ R is transitive.

Q5. Check whether the relation R in R defined as R = {(a, b): a ≤ b³} is reflexive, symmetric or transitive.

A.5. We have,

R=$\{(a,b):a\le b^3\}$ is a relation in R.

For, $(a,b)\in R$ and $a=\frac{1}{2}$ we can write

$a\le a^3 => \frac{1}{2}\le\left(\frac{1}{2}\right)^3 => \frac{1}{2}\le\frac{1}{8}$ which is not true.

So, R is not reflexive.

For $(a,b)=(1,2)\in R$ we have,

$a\le b^3 => 1\le 2^3 => 1\le 8$ is true.

So, $(1,2)\in R$

But $2\le 1^3 => 2\le 1$ is not true

So, $(2,1)\notin R$ and $(b,a)\notin R$

Hence, R is not symmetric.

For, $(a,b)=(10,4)$ and $(b,c)=(4,2)\in R$

$10\le 4^3 => 10\le 64$ is true=> $(10,4)\in R$

$4\le 2^3 => 4\le 8$ is true=> $(4,2)\in R$

But $10\le 2^3 => 10\le 8$ is not true=> $(10,2)\notin R$

Hence, for $(a,b),(b,c)\in R,(a,c)\notin R$

So, R is not transitive.

Q6. Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.

A.6. We have,

$R=\{(1,2),(2,1)\}$ is a relation in set $\{1,2,3\}$

Then, as $(1,1)\notin R$ and $(2,2)\notin R$

So, R is not reflective

As $(1,2)\in R$ and $(2,1)\in R$

So, R is symmetric

And as $(1,2)\in R,(2,1)\in R$ but $(1,1)\notin R$

So, R is not transitive.

Q7. Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y): x and y have same number of pages} is an equivalence relation.

A.7. We have,

$R=\{(x,y): x \,\&\, y \text{ have same number of pages}\}$ is a relation in set of A of all books in

For $(x,y)\in R \,\&\, x,y\in A$

As x=y=same no. of pages

Then, $(x,x)\in R$

Hence, R is reflexive.

For $(x,y)\in R$ and $x,y\in A$

Also, $(y,x)\in R$, $\therefore x=y$

Hence, R is symmetric.

For $x,y,z\in A$ and $(x,y)\in R$ and $(y,z)\in R$

x=y and y=z

⇨ x=z

i.e., $(x,z)\in R$

hence, R is also transitive

$\therefore$ R is an equivalence relation.

Q8. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a,b) : |a-b| is even} is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

A.8. We have,

$R=\{(a,b)\therefore |a-b| \text{ is even}\}$ is a relation in set $A=\{1,2,3,4,5\}$

For all $a\in A, |a-a|=0$ is even.

So, $(a,a)\in R$. Hence R is reflexive

For $a, b \in A$ and $(a, b) \in R$

$|a-b|$ is even

⇨ $|-b+a|$ is even $|$

⇨ $|-(b-a)|$ is even

⇨ $|b-a|$ is even

i.e., $(b, a) \in R$

Hence, R is symmetric.

For $a, b, c \in A$ and $(a, b) \in R$ and $(b, c) \in R$

We have $|a-b|$ is even

and $|b-c|$ is even

then, $|a-b|+|b-c|$ is even as even + even=even

⇨ $|a-b+b-c|$ is even

⇨ $|a-c|$ is even

$\therefore (a, c) \in R$

So, R is transitive.

$\therefore$ R is an equivalence relation

All elements of [1,3,5] are odd positive numbers and its subset are odd and their difference given an even number. Hence, they are related to each other.

Similarly, all elements of [2,4] are even positive numbers and its subset are even and their difference gives an even number. Hence, they are related to each other.

However, elements of [1,3,5] are related to elements of [2,4] since the difference of the two subsets is never even.

Q9. Show that each of the relation R in the set A = {x ∈ Z: 0 ≤ x ≤ 12} ,given by:

(i) R = { (a,b) : |a - b| is a multiple of 4}

(ii) R = { (a,b) : a= b} is an equivalence relation. Find the set of all elements related to 1 in each case.

A.9. We have,

$A=\{x \in 2, 0 \le x \le 12\}$

(i) The relation in set A is defined by
$R= \{(a,b) : |a-b|$ is a multiple of $4\}$

For all $a \in A$,
$|a-a| = 0$ is a multiple of 4

So, $(a,a) \in R$ i.e., R is reflexive
For $a, b \in A \& (a,b) \in R$ we have,
$|a-b|$ is multiple of 4
$|-(b-a)|$ is multiple of 4
$|b-a|$ is multiple of 4
So, $(b,a) \in R$
i.e., R is symmetric

for $a,b,c \in A \ \&(a,b) \in R \&(b,c) \in R$
$|a-b|$ & $|b-c|$ is a multiple of 4
So $|a-b|+|b-c|$ is also a multiple of 4
$|a-b+b-c|$ is a multiple of 4
$|a-c|$ is a multiple of 4
So, $(a,c) \in R$
i.e., R is transitive
Hence, R is an equivalence relation.

Finding all set of elements related to 1
For $a \in A$
Then, $(a,1) \in R$ i.e., $|a-1|$ is a multiple of 4
So, a can be $0 \le a \le 12$
Only,
$|1-1| = 0$
$|5-1| = 4$ is a multiple of 4
$|9-1| = 8$
Hence, sets of elements related to $A=\{1,5,9\}$

(ii) The relation in set A is defined as $R=\{(a,b) : a=b\}$
For all $a \in A$,

a=a is true

so, $(a,a) \in R$

i.e., R is reflexive

for $a,b \in A \& (a,b) \in R$ we have,

a=b

b=a i.e., $(b,a) \in R$

so, R is symmetric

for $a,b,c \in A, (a,b) \in R$ and $(b,c) \in R$. We have,

a=b and b=c

⇨ a=c i.e., $(a,c) \in R$

$\therefore$ R is transitive

Hence, R is an equivalence relation

Find all sets of elements related to 1

For $a \in A$, we need $(a,1) \in R$

i.e., a=1

so, set of elements related to 1 in A is $\{1\}$.

Q10. Given an example of a relation. Which is

(i) Symmetric but neither reflexive nor transitive.

(ii) Transitive but neither reflexive nor symmetric.

(iii) Reflexive and symmetric but not transitive.

(iv) Reflexive and transitive but not symmetric.

(v) Symmetric and transitive but not reflexive.

A.10. Let A= $\{a,b,c\}$

(i) R= $\{(a,b),(b,a)\}$ is a relation in set A

So, $(a,b) \in R$ and $(b,a) \in R \rightarrow$ Symmetric

$(a,a) \notin R \rightarrow$ not reflexive

$(a,b) \in R, (b,a) \in R$ but $(a,a) \notin R \rightarrow$ not transitive

(ii) R= $\{(a,b),(b,c),(a,c)\}$ is a relation in set A

So, $(a,a) \notin R \rightarrow$ not reflexive

$(a,b)\in R$ but $(b,a)\notin R\to$ not symmetric

$(a,b)\in R\,\&\,(b,c)\in R$ and also $(a,c)\in R\to$ transitive

(iii)$R=\{(a,a),(b,b),(c,c),(a,b),(b,a),(a,c),(c,a)\}$

So, $(a,a),(b,b),(c,c)\in R\to$ Reflexive

$(a,b)\in R\to(b,a)\in R\to$ Symmetric

$(a,c)\in R\to(c,a)\in R$

$(b,a)\in R$ and $(a,c)\in R$

But $(b,c)\notin R\to$ not transitive

(iv)$R=\{(a,a),(b,b),(c,c),(a,b),(b,c),(a,c)\}$ is s relation in set A

So, $(a,a),(b,b),(c,c)\in R\to$ reflexive

$(a,b)\,\&\,(b,c)\in R$ so, $(a,c)\in R\to$ transitive

$(a,b)\in R$ but $(b,a)\notin R\to$ not symmetric

(v) $R=\{(a,a),(a,b),(b,a)\}$

So, $(b,b)\notin R\to$ not reflexive

$(a,b)\in R$ and $(b,a)\in R\to$ symmetric

And $(a,b)\in R\,\&\,(b,a)\in R$

and also $(a,a)\in R\to$ transitive

Q11. Show that the relation R in the set A of points in a plane given by R = {(P, Q): distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all point related to a point P ≠ (0, 0) is the circle passing through P with origin as centre.

A.11. The given relation in set A of points in a plane is

$R=\{(P,Q):\text{distance of point P from origin}=\text{distance of point Q from origin}\}$

If O is the point of origin

$R=\{(P,Q):PO=QO\}$

Then, for $P\in A$ we have PO=PO

So, $(P,P)\in R$

i.e., P is reflexive

for, $P,Q\in A$ and $(P,Q)\in R$ we have

⇨ PO=QO

⇨ QO=PO i.e., $(Q,P)\in R$

i.e., R is symmetric

for $P,Q,S\in A$ and $(P,Q)\&(Q,S)\in R$

PO=QO and QO=SO

⇨ PO=SO

i.e., $(P,S)\in R$

so, R is transitive

Hence, R is an equivalence relation

For a point $P\neq(o,o)$ the set of all points related to P i.e., distance from origin to the points are equal is a circle with center at origin (o,o) by the definition of circle

Q12. Show that the relation R defined in the set A of all triangles as R = {(T_1, T_2): T_1 is similar to T_2}, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

A.12. The given relation to set A of all triangles is defined as

$R=\{(T_1,T_2):T_1 \text{ is similar to } T_2\}$

For $T_1\in A$,

T_1 is always similar to T_1

So, $(T_1,T_1)\in R$. Hence R is reflexive.

For $T_1,T_2\in A$ and $(T_1,T_2)\in R$ we have

$T_1\sim T_2(\text{similar})$

⇨ $T_2\sim T_1$ i.e., $(T_2,T_1)\in R$

so, R is symmetric.

for, $T_1,T_2,T_3\in A$ and $(T_1,T_2)\in R$ and $(T_2,T_3)\in R$

$T_1\sim T_2$ and $T_2\sim T_3$

i.e., $T_1\sim T_3\rightarrow(T_1,T_3)\in R$

so, R is transitive

$\therefore$ R is an equivalence relation.

Given, sides of T_1 are 3,4,5

Sides of T_2 are 5,12,13

Sides of T_3 are 6,8,10

As $\frac{3}{5} \neq \frac{4}{12} \neq \frac{5}{13}$ we conclude that T_1 is not similar to T_2

As $\frac{5}{6} \neq \frac{12}{8} \neq \frac{13}{10}$ we conclude that T_2 is not similar to T_3

But as $\frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{1}{2}$ we conclude that $T_1 \sim T_3$

Hence, T_1 is related to T_3

Q13. Show that the relation R defined in the set A of all polygons as R = {(P_1, P_2): P_1 and P_2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right-angle triangle T with sides 3, 4 and 5?

A.13. The given relation in set A of all polygons is defined as

$R=\{(P_1,P_2): P_1 \text{ and } P_2 \text{ have same number of sides}\}$

Let $P_1 \in A$,

As number of sides (P_1) = number of sides (P_1)

$(P_1, P_1) \in R$

So, R is reflexive.

Let $P_1, P_2 \in A$ and $(P_1, P_2) \in R$

Then, number of sides of P_1 = number of sides of P_2

$\Rightarrow$ Number of sides of P_2 = number of sides of P_1

i.e., $(P_2, P_1) \in R$

so, R is symmetric.

Let $P_1, P_2, P_3 \in A$ and (P_1, P_2) and $(P_2, P_3) \in R$

Then, number of sides (P_1) = number of sides (P_2)

Number of sides (P_2)= number of sides (P_3)

So, number of sides (P_1)= number of sides (P_3)

I.e., $(P_1, P_3) \in R$

So, R is transitive.

Hence, R is an equivalence relation.

The set of all triangles will be the elements in A related to the right-angle triangle T with sides 3,4 and 5 as they all have three number of sides.

Q14. Let L be the set of all lines in XY plane and R be the relation in L defined as R = {(L_1, L_2): L_1 is parallel to L_2}. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x + 4.

A.14. The given relation in the set $L =$all lines in $XY -$plane is defined as

$$R = \{(L_1, L_2): L_1 \text{ is parallel to } L_2\}$$

Let $L_1 \in A$ then as L_1 is parallel to L_1,

$(L_1, L_1) \in R$

So, R is reflexive

Let $L_1, L_2 \in A$ and $(L_1, L_1) \in R$

Then, L_1 is parallel to L_2

L_2 is parallel to L_1

So, $(L_2, L_1) \in R$

i.e., R is symmetric

Let $L_1, L_2, L_3 \in A$ and (L_1, L_2) and $(L_2, L_3) \in R$

Then, $L_1 \| L_2$ and $L_2 \| L_3$

So, $L_1 \| L_3$

i.e., $(L_1, L_2) \in R$

So, R is transitive

Hence, R is an equivalence relation

The set of lines related to $y=2x+4$ is given by the equation $y=2x+C$ where C is some constant.

Q15. Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer.

(A) R is reflexive and symmetric but not transitive.

(B) R is reflexive and transitive but not symmetric.

(C) R is symmetric and transitive but not reflexive.

(D) R is an equivalence relation.

A.15. The set in $A=\{1,2,3,4\}$

The relation in this set A is given by

$$R=\{(1,2),\ (2,2),\ (1,1),\ (4,4),\ (1,3),\ (3,3),\ (3,2)\}$$

R is reflexive as $(1,1),\ (2,2),\ (3,3),\ (4,4)\ \in R$

As, $(1,2)\ \in R$ but $(2,1)\ \notin R$

R is not symmetric

For $(1,2)\ \in R$ and $(2,2)\ \in R$; $(1,2)\ \in R$

And for $(1,3)\ \in R$ and $(3,2)\ \in R$; $(1,3)\ \in R$

$\therefore$ R is transitive

Hence, option (B) is correct

Q16. Let R be the relation in the set N given by R = {(a, b): a = b − 2, b > 6}. Choose the correct answer.

(A) (2, 4) ∈ R (B) (3, 8) ∈R (C) (6, 8) ∈R (D) (8, 7) ∈ R

A.16. The given relation in set N defined by

$$R=\{(a,b)\ :\ a=b-2,\ b>6\}$$

For (2,4), 4>6 is not true

For (3,8), 8>6 but 3= 8-2 ⇒ 3=6 is not true

For (6,8), 8>6 and 6= 8-2 ⇒ 6=6 is true

And for (8,7), 7>6 but 8= 7-2 ⇒ 8=5 is not true

Hence, option (C) is correct