Ex-9.2

In each of the Questions 1 to 6 verify that the given functions (explicit) is a solution of the corresponding differential equation:

Q1. $y = e^x + 1 \; : \; y'' - y' = 0$

A.1. Given fx^n is $y = e^x + 1$

Differentiating with x we get,

$$y^{|} = \frac{dy}{dx} = e^x$$

Again,

$$y^{||} = \frac{d^2y}{dx^2} = e^x$$

Substituting value of y'' and y' in the given D.E. we get

$L.H.S. = y'' - y' = e^x - e^x = 0 = R.H.S$

$\therefore$ The given fx^n is a solution of the given D.E.

Q2. $y = x^2 + 2x + C$: $y' - 2x - 2 = 0$

A.2. Given , fx^n is $y = x^2 + 2x + c$

So, $y' = 2x + 2$

Substituting value of y' in the given D.E. we get,

$L.H.S. = y' - 2x - 2 = 2x + 2 - 2x - 2 = 0 = R.H.S$

$\therefore$ The given fx^n is a solution of the given D.E.

Q3. $y = \cos x + C$: $y' + \sin x = 0$

A.3. Given, fx^n is $y = \cos x + c$

So, $y' = -\sin x$

Putting the value of y' in the given D.E. we get,

$L.H.S. = y' + \sin x = -\sin x + \sin x = 0 = R.H.S$

$\therefore$ The given fx^n is a solution of the given D.E.

Q4. $y = \sqrt{1 + x^2}$; $y' = \dfrac{xy}{1+x^2}$

A.4. Given, $y = \sqrt{1+x^2}$

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So, $y^{|} = \frac{1}{2\sqrt{1+x^2}}\frac{d}{dx}\left(1+x^2\right) = \frac{2x}{2\sqrt{1+x^2}} = \frac{x}{1+x^2}$

R.H.S. of the given D.E= $\frac{xy}{1+x^2}$

$$= \frac{x}{1+x^2} \times \sqrt{1+x^2}$$
$$= \frac{x}{\sqrt{1+x^2}}$$
$$= y^{|} = L.H.S$$

Hence, the given fx^n is solution of the given D.E.

Q5. y = Ax : xy′ = y (x ≠ 0)

A.5. Given, $y = Ax:$

So, $y^{|} = \frac{Adx}{dx} = A$

Putting value of $y^{|}$ in L.H.S. of the given D.E.

L.H.S= $xy^{|} = xA = Ax = y$=R.H.S

$\therefore$ The given fx^n is a solution of the given D.E.

Q6. $y = x\sin x : xy' = y + x\sqrt{x^2 - y^2}\ (x \neq 0 \text{ and } x > y, \text{ or } x < -y)$

A.6. Given, $y = x\sin x$

So, $y^{|} = x\frac{d}{dx}\sin x + \sin x\frac{dx}{dx} = x\cos x + \sin x$

Now, L.H.S of the given D.E $= xy^{|}$

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$$= x(x\cos x + \sin x)$$

$$= x^2 \cos x + x\sin x$$

And R.H.S. of the given D.E $= y + x\sqrt{x^2 - y^2}$

$$= x\sin x + \sqrt{x^2 - (x\sin x)^2}$$
$$= x\sin x + x\sqrt{x^2 - x^2\sin^2 x}$$
$$= x\sin x + x\sqrt{x^2 - (1 - \sin^2 x)} \left\{\because 1 = \cos^2 x + \sin^2\right\}$$
$$= x\sin x + x^2\sqrt{\cos^2}$$
$$= x\sin x + x^2 \cos x$$
$$= x^2 \cos x + x\sin x$$

Hence, L.H.S =R.H.S

Therefore, the given fx^n is a solution of the given D.E.

Q7. xy = log y + C : $y' = \frac{y^2}{1 - xy}(xy \neq 1)$

A.7. Given, $xy = \log y + c$

Differentiate w.r.t. x we have

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$$x\frac{dy}{dx}+y\frac{dx}{dx}=\frac{d}{dx}\log y+\frac{d}{dx}C$$
$$\Rightarrow x\frac{dy}{dx}+y=\frac{1}{y}\frac{dy}{dx}+0$$
$$\Rightarrow x\frac{dy}{dx}-\frac{1}{y}\frac{dy}{dx}=-y$$
$$\Rightarrow \frac{dy}{dx}\left[x-\frac{1}{y}\right]=-y$$
$$\Rightarrow \frac{dy}{dx}\left[\frac{xy-1}{y}\right]=-y$$
$$\Rightarrow \frac{dy}{dx}=\frac{-y^2}{xy-1}=\frac{(-1)\times -y^2}{(-1)\times(xy-1)}=\frac{y^2}{1-xy}$$
$$\therefore y^{|}=\frac{y^2}{1-xy}$$

Hence, y is a Solution of the given D.E

Q8. y – cos y = x : (y sin y + cos y + x) y′ = y

A.8. Given $y-\cos y=x$

Differentiate w.r.t 'x' we get

$$\frac{dy}{dx}-(-\sin y)\frac{dy}{dx}=\frac{dx}{dx}$$
$$\Rightarrow \frac{dy}{dx}[1+\sin y]=1$$
$$\Rightarrow \frac{dy}{dx}=\frac{1}{1+\sin y}=y^{|}$$

So, L.H.S of given D.E $=(y\sin y+\cos y+x)y^{|}$

$$=(y\sin y+\cos y+y-\cos x)\left[\frac{1}{1+\sin y}\right]$$
$$=\frac{y(1+\sin y)}{(1+\sin y)}=y=R.H.S$$

$\therefore$ The given fx^n is a solution of the given D.E.

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Q9. $\mathbf{x + y = tan^{-1} y \quad : \quad y^2y' + y^2 + 1 = 0}$

A.9. Given, $x + y = \tan^{-1} y$

Differentiate with 'x' we get

$$1+\frac{dy}{dx}=\frac{1}{1+y^2}\frac{dy}{dx}$$

$$=1+y^{|}=\frac{1}{1+y^2}y^{|}$$

$$=\left(1+y^{|}\right)\left(1+y^2\right)=y^{|}$$

$$=1+y^2y^{|}+y^{|}+y^2=y^{|}$$

$$=y^2y^{|}+y^2+1=0$$

$\therefore$ The given fx^n is a solution of the given D.E

Q10. $y=\sqrt{a^2-x^2x}\in(-a,a) \ : \ x+y\frac{dy}{dx}=0(y\neq 0)$

A.10. Given, $y=\sqrt{a^2-x^2}, x\in(-a,a)$

Differentiate with 'x' we get,

$$\frac{dy}{dx}=\frac{1}{2\sqrt{a^2-x^2}}\frac{d}{dx}\left(a^2-x^2\right)=-\frac{2x}{2\sqrt{a^2-x^2}}$$

$$\frac{dy}{dx}=\frac{-x}{\sqrt{a^2-x^2}}$$

The L.H.S of the given D.E $=x+y\frac{dy}{dx}$

$$= x + \sqrt{a^2 - x^2} \times \left(\frac{-x}{\sqrt{a^2 - x^2}} \right)$$

$$= x - x$$

$$= 0$$

$$= R.H.S$$

$\therefore$ The given fx^n is a solution of the given D.E.

Q11. The number of arbitrary constants in the general solution of a differential equation of fourth order are:

(A) 0

(B) 2

(C) 3

(D) 4

A.11. The number of arbitrary constant is general solution of D.E of 4th order is four.

$\therefore$ Option (D) is correct.

Q12. The number of arbitrary constants in the particular solution of a differential equation of third order are:

(A) 3

(B) 2

(C) 1

(D) 0

A.12. In a particular solution, there are no arbitrary constant.