Q1. For each of the differential equations given below, indicate its order and degree (if defined):

$$(i)\ \frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2-6y=\log x$$

$$(ii)\ \left(\frac{dy}{dx}\right)^3-4\left(\frac{dy}{dx}\right)^2+7y=\sin x$$

$$(iii)\ \frac{d^4y}{dx^4}-\sin\frac{d^3y}{dx^3}=0$$

A.1.

(i) Given: Differential equation $\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2-6y=\log x$

The highest order derivative present in this differential equation is $\frac{d^2y}{dx^2}$ and hence order of this differential equation if 2.

The given differential equation is a polynomial equation in derivatives and highest power of the highest order derivative $\frac{d^2y}{dx^2}$ is 1.

Therefore, Order = 2, Degree = 1

(ii) Given: Differential equation $\left(\frac{dy}{dx}\right)^3 - 4\left(\frac{dy}{dx}\right)^2 + 7y = \sin x$

The highest order derivative present in this differential equation is $\frac{dy}{dx}$ and hence order of this differential equation if 1.

The given differential equation is a polynomial equation in derivatives and highest power of the highest order derivative $\frac{dy}{dx}$ is 3.

Therefore, Order = 1, Degree = 3

(iii) Given: Differential equation $\frac{d^4y}{dx^4} - \sin\frac{d^3y}{dx^3} = 0$

The highest order derivative present in this differential equation is $\frac{d^4y}{dx^4}$ and hence order of this differential equation if 4.

The given differential equation is not a polynomial equation in derivatives therefore, degree of this differential equation is not defined.

Therefore, Order = 4, Degree not defined

Q2. For each of the exercises given below verify that the given function (implicit or explicit) is a solution of the corresponding differential equation:

$$(i) y a e^{2}+b e^{-x}+x^{2}: x \frac{d^{2} y}{d x^{2}}+2 \frac{d y}{d x}-x y+x^{2}-2=0$$

$$(ii) y=e^{x}(a \cos x+b \sin x): \frac{d^{2} y}{d x^{2}}-2 \frac{d y}{d x}+2 y=0$$

$$(iii) y=x \sin 3 x: \frac{d^{2} y}{d x^{2}}+9 y-6 \cos 3 x=0$$

$$(iv) x^{2}=2 y^{2} \log y:(x^{2}+y^{2}) \frac{d y}{d x}-x y=0$$

A.2.

(i) $y a e^{2}+b e^{-x}+x^{2}$

Differentiating both sides with respect to x, we get:

$$\frac{d y}{d x}=a \frac{d}{d x}\left(e^{x}\right)+b \frac{d}{d x}\left(e^{-x}\right)+\frac{d}{d x}\left(x^{2}\right)$$

$$\Rightarrow \frac{d y}{d x}=a e^{x}-b e^{-x}+2 x$$

Again, differentiating both sides with respect to x, we get:

$$\frac{d^{2} y}{d x^{2}}=a e^{x}-b e^{-x}+2 x$$

Now, on substituting the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in the differential equation, we get:

L.H.S

$$x \frac{d^{2} y}{d x^{2}}+2 \frac{d y}{d x}-x y+x^{2}-2$$

$$=x\left(a e^{x}-b e^{-x}+2\right)+2\left(a e^{x}-b e^{-x}+2 x\right)-x\left(a e^{x}+b e^{-x}+x^{2}\right)+x^{2}-2$$

$$=\left(x a e^{x}-b x e^{-x}+2 x\right)+\left(2 a e^{x}-2 b e^{-x}+4 x\right)-\left(a x e^{x}+b x e^{-x}+x^{3}\right)+x^{2}-2$$

$$=2 a e^{x}-2 b e^{-x}+x^{2}+6 x-2$$

$$\neq 0$$

Therefore, Function given by equation (i) is a solution of differential equation. (ii).

(ii) $y=e^{x}(a \cos x+b \sin x)=a e^{x} \cos x+b e^{x} \sin x$

Differentiating both sides with respect to x, we get:

$$\frac{dy}{dx} = a.\frac{d}{dx}(e^x \cos x) + b.\frac{d}{dx}(e^x \sin x)$$
$$\Rightarrow \frac{dy}{dx} = a\left(e^x \cos x - e^x \sin x\right) + b.\left(e^x \sin x + e^x \cos x\right)$$
$$\Rightarrow \frac{dy}{dx} = \left(a+b\right)e^x \cos x + (b-a)e^x \sin x$$

Again, differentiating both sides with respect to x, we get:

$$\frac{d^2y}{dx^2} = \left(a+b\right).\frac{d}{dx}(e^x \cos x)(b-a)\frac{d}{dx}(e^x \sin x)$$
$$\Rightarrow \frac{d^2y}{dx^2} = \left(a+b\right).\left[e^x \cos x - e^x \sin x\right] + (b-a)\left[e^x \sin x + e^x \cos x\right]$$
$$\Rightarrow \frac{d^2y}{dx^2} = e^x\left[\left(a+b\right)(\cos x - \sin x) + (b-a)(\sin x + \cos x)\right]$$
$$\Rightarrow \frac{d^2y}{dx^2} = e^x\left[a\cos x - a\sin x + b\cos x - b\sin x + b\sin x + b\cos x - a\sin x - a\cos x\right]$$
$$\Rightarrow \frac{d^2y}{dx^2} = \left[2e^x(b\cos x - a\sin x)\right]$$

Now, on substituting the values of $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$ in the L.H.S of the given differential equation, we get:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y$$
$$= 2e^x(b\cos x - a\sin x) - 2e^x\left[\left(a+b\right)\cos x + (b-a)\sin x\right] + 2e^x(a\cos x + b\sin x)$$
$$= e^x\left[(2b\cos x - 2a\sin x) - (2a\cos x + 2b\cos x) - (2b\sin x - 2a\sin x) + (2a\cos x + 2b\sin x)\right]$$
$$= e^x\left[\left(2b - 2a - 2b + 2a\right)\cos x\right] + e^x\left[\left(-2a - 2b + 2a + 2b\right)\sin x\right]$$
$$= 0$$

Therefore, Function given by equation (i) is solution of differential equation (ii)

(iii) $y = x\sin 3x$

Differentiating both sides with respect to x, we get:

$$\frac{dy}{dx} = \frac{d}{dx}(xsin3x) = \sin 3x + x.\cos 3x.3$$
$$\Rightarrow \frac{dy}{dx} = \sin 3x + 3x\cos 3x$$

Again, differentiating both sides with respect to x, we get:

$$\frac{d^2y}{dx^2}=\frac{d}{dx}(\sin 3x)+3\frac{d}{dx}(x\cos 3x)$$
$$\Rightarrow \frac{d^2y}{dx^2}=3\cos 3x+3[\cos 3x+x(-\sin 3x).3]$$
$$\Rightarrow \frac{d^2y}{dx^2}6\cos 3x-9x\sin 3x$$

Substituting the value of $\frac{d^2y}{dx^2}$ in the L.H.S. of the given differential equation, we get:

$$\frac{d^2y}{dx^2}+9y-6\cos 3x$$
$$=(6.\cos 3x-9x\sin 3x)+9x\sin 3x-6\cos 3x$$
$$=0$$

Therefore, Function given by equation (i) is a solution of differential equation (ii).

(iv) $x^2=2y^2\log y$

Differentiating both sides with respect to x, we get:

$$2x=2.\frac{d}{dx}=\left[y^2\log y\right]$$
$$\Rightarrow x=\left[2y.\log y.\frac{dy}{dx}+y^2.\frac{1}{y}.\frac{dy}{dx}\right]$$
$$\Rightarrow x=\frac{dy}{dx}(2y\log y+y)$$
$$\Rightarrow \frac{dy}{dx}=\frac{x}{y(1+2\log y)}$$

Substituting the value of $\frac{dy}{dx}$ in the L.H.S. of the given differential equation, we get:

$$\left(x^2+y^2\right)\frac{dy}{dx}-xy$$
$$=\left(2y^2\log y+y^2\right).\frac{x}{y\left(1+2\log y\right)}-xy$$
$$=y^2(1+2\log y).\frac{x}{y\left(1+2\log y\right)}-xy$$
$$=xy-xy$$
$$=0$$

Therefore, Function given by equation (i) is a solution of differential equation (ii).

Q3. Form the differential equation representing the family of curves $(x-a)^2+2y^2=a^2$ **where a an arbitrary constant**.

A.3. Equation of the given family of curves is $(x-a)^2+2y^2=a^2$

$$(x-a)^2+2y^2=a^2$$
$$\Rightarrow x^2+a^2-2ax+2y^2=a^2$$
$$\Rightarrow 2y^2=2ax-x^2 \text{..........}(1)$$

Differentiating with respect to x, we get:

$$2y\frac{dy}{dx}=\frac{2a-2x}{2}$$
$$\Rightarrow \frac{dy}{dx}=\frac{a-x}{2y}$$
$$\Rightarrow \frac{dy}{dx}=\frac{2a-2x^2}{4xy} \text{..........}(2)$$

From equation (*1), we get:

$$2ax=2y^2+x^2$$

On substituting this value in equation (3), we get:

$$\frac{dy}{dx}=\frac{2y^2+x^2-2x^2}{4xy}$$
$$\Rightarrow \frac{dy}{dx}=\frac{2y^2-x^2}{4xy}$$

Hence, the differential equation of the family of curves is given as $\frac{dy}{dx}=\frac{2y^2-x^2}{4xy}$.

Q4. Prove that $x^2 - y^2 = c\left(x^2 + y^2\right)^2$ is the general equation of the differential equation $\left(x^3 - 3xy^2\right)dx = \left(y^3 - 3x^2y\right)dy$ where c is a parameter.

A.4.

$$\Rightarrow \frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y} \text{..........(1)}$$

This is a homogenous equation. To simplify it, we need to make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting the values of y and $\frac{dv}{dx}$ in equation (1), we get:

$$v + x\frac{dv}{dx} = \frac{x^3 - 3x(vx)^2}{(vx)^3 - 3x^2(vx)}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1 - 3v^2}{v^3 - 3v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 - 3v^2}{v^3 - 3v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 - 3v^2 - v(v^3 - 3v)}{v^3 - 3v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 - v^4}{v^3 - 3v}$$

$$\Rightarrow \left(\frac{v^3 - 3v}{1 - v^4}\right)dv = \frac{dx}{x}$$

Integrating both sides, we get:

$$\int\left(\frac{v^3 - 3v}{1 - v^4}\right)dv = \log x + \log C' \text{.........(2)}$$

$$Now, \int\left(\frac{v^3 - 3v}{1 - v^4}\right)dv = \int\frac{v^3dv}{1 - v^4} - 3\int\frac{vdv}{1 - v^4}$$

$$\Rightarrow \int\left(\frac{v^3 - 3v}{1 - v^4}\right)dv = I_1 - 3I_2, Where, I_1 = \int\frac{v^3dv}{1 - v^4} and I_2 = \int\frac{vdv}{1 - v^4} \text{...........(3)}$$

$Let, 1-v^4 = t.$

$\therefore \frac{d}{dv}(1-v^4) = \frac{dt}{dv}$

$\Rightarrow -4v^3 = \frac{dt}{dv}$

$\Rightarrow v^3 dv = -\frac{dt}{4}$

$Now, I_1 = \int -\frac{dt}{4} = -\log t = -\frac{1}{4}\log(1-v^4)$

$And, I_2 = \int \frac{vdv}{1-v^4} = \int \frac{vdv}{1-(v^2)^2}$

$Let, v^2 = p.$

$\therefore \frac{d}{dv}\left(v^2\right) = \frac{dp}{dv}$

$\Rightarrow 2v = \frac{dp}{dv}$

$\Rightarrow vdv = \frac{p}{2}$

$\Rightarrow I_2 = \frac{1}{2}\int \frac{dp}{1-p^2} = \frac{1}{2\times 2}\log\left|\frac{1+p}{1-p}\right| = \frac{1}{4}\log\left|\frac{1+v^2}{1-v^2}\right|$

Substituting the values of I_1 and I_2 in equation (3), we get:

$$\int\left(\frac{v^3-3v}{1-v^4}\right)dv = -\frac{1}{4}\log(1-v^4) - \frac{3}{4}\log\left|\frac{1+v^2}{1-v^2}\right|$$

Therefore, equation (2) becomes:

$$\frac{1}{4}\log(1-v^4)-\frac{3}{4}\log\left|\frac{1+v^2}{1-v^2}\right|=\log x+\log C'$$

$$\Rightarrow -\frac{1}{4}\log\left[(1-v^4)\left(\frac{1+v^2}{1-v^2}\right)\right]=\log C'x$$

$$\Rightarrow \frac{\left(1+v^2\right)^4}{\left(1-v^2\right)^2}=\left(C'x\right)^{-4}$$

$$\Rightarrow \frac{\left(1+\frac{y^2}{x^2}\right)^4}{\left(1-\frac{y^2}{x^2}\right)^2}=\frac{1}{C'^4x^4}$$

$$\Rightarrow \frac{\left(x^2+y^2\right)^4}{x^4\left(x^2-y^2\right)^2}=\frac{1}{C'^4x^4}$$

$$\Rightarrow \left(x^2-y^2\right)^2=C'^4\left(x^2+y^2\right)^4$$

$$\Rightarrow \left(x^2-y^2\right)=C'^2\left(x^2+y^2\right)^2$$

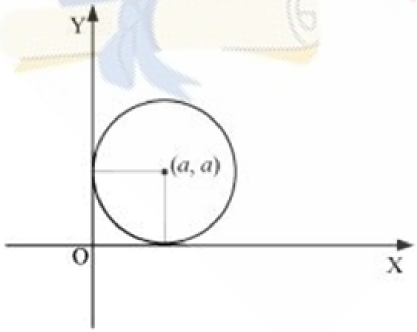
$$\Rightarrow x^2-y^2=C\left(x^2+y^2\right)^2, where C=C'^2$$

Hence, the given result is proved.

Q5. For the differential equation of the family of the circles in the first quadrant which touch the coordinate axes.

A.5. The equation of a circle in the first quadrant with centre (a, a) and radius (a) which touches the coordinate axes is:

$$(x-a)^2+(y-a)^2=a^2..........(1)$$



Differentiating equation (1) with respect to x, we get:

$$2(x-a)+2(y-a)\frac{dy}{dx}=0$$
$$\Rightarrow (x-a)+(y-a)y'=0$$
$$\Rightarrow x-a+yy'-ay'=0$$
$$\Rightarrow x+yy'-a(1+y')=0$$
$$\Rightarrow a=\frac{x+yy'}{1+y'}$$

Substituting the value of a in equation (1), we get:

$$\left[x-\left(\frac{x+yy'}{1+y'}\right)\right]^2+\left[y-\left(\frac{x+yy'}{1+y'}\right)\right]^2=\left(\frac{x+yy'}{1+y'}\right)^2$$
$$\Rightarrow \left[\frac{(x-a)y'}{(1+y')}\right]^2+\left[\frac{y-x}{1+y'}\right]^2=\left[\frac{x+yy'}{1+y'}\right]^2$$
$$\Rightarrow (x-y)^2.y'^2+(x-y)^2=(x+yy')^2$$
$$\Rightarrow (x-y)^2\left[1+(y')^2\right]=(x+yy')^2$$

Hence, the required differential equation of the family of circles is $(x-y)^2\left[1+(y')^2\right]=(x+yy')^2$

Q6. Find the general solution of the differential equation

$$\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$$

A6.

Given: Differential Equation $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$

$$\Rightarrow \frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$$
$$\Rightarrow \frac{dy}{\sqrt{1-y^2}}=\frac{-dx}{\sqrt{1-x^2}}$$

Integrating both sides, we get:

$$\sin^{-1}y=-\sin^{-1}x+C$$
$$\Rightarrow \sin^{-1}x+\sin^{-1}y=C$$

Q7. Show that the general solution of the differential equation $\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1}=0$ **is given by** $(x+y+1)A(1-x-y-2xy),$ **where A is parameter.**

A.7.

Given: Differential equation $\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1}=0$

$$\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1}=0$$
$$\Rightarrow \frac{dy}{dx}=-\frac{(y^2+y+1)}{x^2+x+1}$$
$$\Rightarrow \frac{dy}{y^2+y+1}=\frac{-dx}{x^2+x+1}$$
$$\Rightarrow \frac{dy}{y^2+y+1}+\frac{dx}{x^2+x+1}=0$$

Integrating both sides,

$$\int\frac{dy}{y^2+y+1}+\int\frac{dx}{x^2+x+1}=C$$
$$\Rightarrow \int\frac{dy}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}+\int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=C$$
$$\Rightarrow \frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{y+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right]+\frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right]=C$$
$$\Rightarrow \tan^{-1}\left[\frac{2y+1}{\sqrt{3}}\right]+\tan^{-1}\left[\frac{2x+1}{\sqrt{3}}\right]=\frac{\sqrt{3}C}{2}$$
$$\Rightarrow \tan^{-1}\left[\frac{\frac{2y+1}{\sqrt{3}}+\frac{2x+1}{\sqrt{3}}}{1-\frac{(2y+1)}{\sqrt{3}}.\frac{(2x+1)}{\sqrt{3}}}\right]=\frac{\sqrt{3}C}{2}$$
$$\Rightarrow \tan^{-1}\left[\frac{\frac{2x+2y+2}{\sqrt{3}}}{1-\left(\frac{4xy+2x+2y+1}{3}\right)}\right]=\frac{\sqrt{3}C}{2}$$
$$\Rightarrow \tan^{-1}\left[\frac{2\sqrt{3}(x+y+1)}{3-4xy-2x-2y-1}\right]=\frac{\sqrt{3}C}{2}$$

$$\Rightarrow \tan^{-1}\left[\frac{\sqrt{3}(x+y+1)}{2(1-x-y-2xy)}\right]=\frac{\sqrt{3}C}{2}$$

$$\Rightarrow \frac{\sqrt{3}(x+y+1)}{2(1-x-y-2xy)}=\tan\left(\frac{\sqrt{3}C}{2}\right)=B, where, B=\tan\left(\frac{\sqrt{3}C}{2}\right)$$

$$\Rightarrow x+y+1=\frac{2B}{\sqrt{3}}(1-xy-2xy)$$

$$\Rightarrow x+y+1=A(1-x-y-2xy), where, A=\frac{2B}{\sqrt{3}}$$

Q8. Find the equation of the curve passing through the point (0,π/4), whose differential equation is sin x cos y dx + cos x sin y dy = 0.

A.8. The differential equation of the given curve is:

$$\sin x\cos y dx+\cos x\sin y dy=0$$

$$\Rightarrow \frac{\sin x\cos y dx+\cos x\sin y dy}{\cos x\cos y}=0$$

$$\Rightarrow \tan x dx+\tan y dy=0$$

Integrating both sides, we get:

$$\log(\sec x)+\log(\sec y)=\log C$$

$$\log(\sec x.\sec y)=\log C$$

$$\Rightarrow \sec x.\sec y=C..........(1)$$

The curve passes through point $\left(0,\frac{\pi}{4}\right)$

$$\therefore 1\times\sqrt{2}=C$$

$$\Rightarrow C=\sqrt{2}$$

On subtracting $C=\sqrt{2}$ in equation (10, we get:

$$\sec x.\sec y=\sqrt{2}$$

$$\Rightarrow \sec x.\frac{1}{\cos y}=\sqrt{2}$$

$$\Rightarrow \cos y=\frac{\sec x}{\sqrt{2}}$$

Q9. Find the particular solution of the differential equation (1 + e2x) dy + (1 + y2) ex dx = 0, given that y = 1 when x = 0.

A.9.

$$\left(1+e^{2x}\right)dy+\left(1+y^2\right)e^x dx=0$$

$$\Rightarrow \frac{dy}{1+y^2}+\frac{e^x dx}{1+e^{2x}}=0$$

Integrating both sides, we get:

$$\tan^{-1} y + \int \frac{e^x dx}{1+e^{2x}} = C..........(1)$$

$$Let, e^x = t \Rightarrow e^{2x} = t^2$$

$$\Rightarrow \frac{d}{dx}\left(e^x\right) = \frac{dt}{dx}$$

$$\Rightarrow e^x = \frac{dt}{dx}$$

$$\Rightarrow e^x dx = dt$$

Substituting these values in equation (1), we get:

$$\tan^{-1} y + \int \frac{dt}{1+t^2} = C$$

$$\Rightarrow \tan^{-1} y + \tan^{-1} t = C$$

$$\Rightarrow \tan^{-1} y + \tan^{-1}(e^x) = C..........(2)$$

$$Now, y = 1, at, x = 0$$

Therefore, equation (2) becomes:

$$\tan^{-1} 1 + \tan^{-1} 1 = C$$

$$\Rightarrow \frac{\pi}{4} + \frac{\pi}{4} = C$$

$$\Rightarrow C = \frac{\pi}{2}$$

Substituting $C = \frac{\pi}{2}$ in equation (2), we get:

$$\tan^{-1} y + \tan^{-1}(e^x) = \frac{\pi}{2}$$

This is the required solution of the given differential equation.

Q10. Solve the differential equation: $ye^{\frac{x}{y}} dx = \left(xe^{\frac{x}{y}} + y^2\right) dy (y \neq 0)$

A.10.

$$ye^{\frac{x}{y}} dx = \left(xe^{\frac{x}{y}} + y^2\right) dy$$

$$\Rightarrow ye^{\frac{x}{y}} \frac{dx}{dy} = xe^{\frac{x}{y}} + y^2$$

$$\Rightarrow e^{\frac{x}{y}} \left[y.\frac{dx}{dy} - x\right] = y^2$$

$$\Rightarrow e^{\frac{x}{y}} . \frac{\left[y.\frac{dx}{dy} - x\right]}{y^2} = 1..........(1)$$

$Let, e^{\frac{x}{y}} = z$

Differentiating it with respect to y, we get:

$$\left(e^{\frac{x}{y}}\right) = \frac{dz}{dy}$$

$$\Rightarrow e^{\frac{x}{y}}.\frac{d}{dy}\left(\frac{x}{y}\right) = \frac{dz}{dy}$$

$$\Rightarrow e^{\frac{x}{y}}.\left[\frac{y.\frac{dx}{dy} - x}{y^2}\right] = \frac{dz}{dy}..........(2)$$

From equation (1) and equation (2), we get:

$$\frac{dz}{dy} = 1$$

$$\Rightarrow dz = dy$$

Integration both sides, we get:

$$z = y + C$$

$$\Rightarrow e^{\frac{x}{y}} y + C$$

Q11. Find a particular solution of the differential equation (x – y) (dx + dy) = dx – dy, given that y = –1, when x = 0. (Hint: put x – y = t)

A.11.

$$(x - y)(dx + dy) = dx - dy$$

$$\Rightarrow (x - y + 1)dy = (1 - x + y)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{1 - x + y}{x - y + 1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1 - (x - y)}{1 + (x - y)}..........(1)$$

$$Let, x - y = t$$

$$\Rightarrow \frac{d}{dx}(x - y) = \frac{dt}{dx}$$

$$\Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow 1 - \frac{dt}{dx} = \frac{dy}{dx}$$

Substituting the values of $x - y$ and $\frac{dy}{dx}$ in equation (1), we get:

$$1-\frac{dt}{dx}=\frac{1-t}{1+t}$$

$$\Rightarrow \frac{dt}{dx}=1-\left(\frac{1-t}{1+t}\right)$$

$$\Rightarrow \frac{dt}{dx}=\frac{(1+t)-(1-t)}{1+t}$$

$$\Rightarrow \frac{dt}{dx}=\frac{2t}{1+t}$$

$$\Rightarrow \left(\frac{1+t}{t}dt\right)=2dx$$

$$\Rightarrow \left(1+\frac{1}{t}\right)dt=2dx..........(2)$$

Integrating both sides, we get:

$$t+\log|t|=2x+C$$

$$\Rightarrow (x-y)+\log|x-y|=2x+C$$

$$\Rightarrow \log|x-y|=x+y+C..........(3)$$

$Now, y=-1, at, x=0$

Therefore, equation (3) becomes:

$$\log 1=0-1+C$$

$$\Rightarrow C=1$$

Substituting $C=1$ in equation (3), we get:

$$og|x-y|=x+y+1$$

This is the required particular solution of the given differential equation .

Q12. Solve the differential equation: $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right]\frac{dx}{dy}=1(x\neq 0)$

A.12.

$$\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right]\frac{dx}{dy}=1$$

$$\Rightarrow \frac{dy}{dx}=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx}+\frac{y}{\sqrt{x}}=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

This equation is a linear differential equation of the form

$$\frac{dy}{dx}+Py=Q, where, P=\frac{1}{\sqrt{x}}\ \&\ Q=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

$$Now, I.F=e^{\int Pdx}=e^{\int \frac{1}{\sqrt{x}}dx}=e^{2\sqrt{x}}$$

The general solution of the given differential equation is given by,

$$y(I.F) = \int (Q \times I.F.)dx + C$$

$$\Rightarrow ye^{2\sqrt{x}} = \int \left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} \times e^{2\sqrt{x}} \right) dx + C$$

$$\Rightarrow ye^{2\sqrt{x}} = \int \frac{1}{\sqrt{x}} dx + C$$

$$\Rightarrow ye^{2\sqrt{x}} = 2\sqrt{x} + C$$

Q13. Find the particular solution of the differential equation $\frac{dy}{dx} + y\cot x = 4x\cos ecx (x \neq 0)$ **given that y = 0 when x = π/2**

A.13.

The given differential equation is:

$$\frac{dy}{dx} + y\cot x = 4x\cos ecx$$

This equation is a linear equation of the form

$$\frac{dy}{dx} + Py = Q, where, p = \cot x \,\&\, Q = 4x\cos ecx$$

$$Now, I.F = e^{\int Pdx} = e^{\int \cot x dx} = e^{\log|\sin x|} = \sin x$$

The general solution of the given differential equation is given by,

$$y(I.F) = \int (Q \times I.F.)dx + C$$

$$\Rightarrow y\sin x = \int (4x\cos ecx.\sin x)dx + C$$

$$\Rightarrow y\sin x = 4\int xdx + C$$

$$\Rightarrow y\sin x = 4.\frac{x^2}{2} + C$$

$$\Rightarrow y\sin x = 2x^2 + C..........(1)$$

$$Now, y = 0 at, x = \frac{\pi}{2}$$

Therefore, equation (1) becomes:

$$0 = 2 \times \frac{\pi}{2} + C$$

$$\Rightarrow C = -\frac{\pi^2}{2}$$

Substituting $C = -\frac{\pi^2}{2}$ in equation (1), we get:

$$y\sin x = 2x^2 - \frac{\pi^2}{2}$$

This is the required particular solution of the given differential equation.

Q14. Find the particular solution of the differential equation $(x+1)\frac{dy}{dx}=2e^{-y}-1$ **given that y = 0 when x = 0**

A.14.

$$(x+1)\frac{dy}{dx}=2e^{-y}-1$$

$$\Rightarrow \frac{dy}{2e^{-y}-1}=\frac{dx}{x+1}$$

$$\Rightarrow \frac{e^{y}dy}{2-e^{y}}=\frac{dx}{x+1}$$

Integrating both sides, we get:

$$\int\frac{e^{y}dy}{2-e^{y}}=\log|x+1|+\log C..........(1)$$

$$Let 2-e^{y}=t$$

$$\therefore \frac{d}{dy}(2-e^{y})=\frac{dt}{dy}$$

$$\Rightarrow -e^{y}=\frac{dt}{dy}$$

$$\Rightarrow e^{y}dy=-dt$$

Substituting this value in equation (1), we get:

$$\int\frac{-dt}{t}=\log og|x+1|+\log C$$

$$\Rightarrow -\log|t|=\log|C(x+1)|$$

$$\Rightarrow -\log|2-e^{y}|=\log|C(x+1)|$$

$$\Rightarrow \frac{1}{2-e^{y}}=C(x+1)$$

$$\Rightarrow 2-e^{y}=\frac{1}{C(x+1)}..........(2)$$

Now, at x=0& y=0, equation (2) becomes:

$$\Rightarrow 2-1=\frac{1}{C}$$

$$\Rightarrow C=1$$

Substituting $C=1$ in equation (2), we get:

$$2-e^{y}=\frac{1}{x+1}$$

$$\Rightarrow e^{y}=2-\frac{1}{x+1}$$

$$\Rightarrow e^{y}=\frac{2x+2-1}{x+1}$$

$$\Rightarrow e^{y}=\frac{2x+1}{x+1}$$

$$\Rightarrow y=\log\left|\frac{2x+1}{x+1}\right|,(x\neq -1)$$

This is the required particular solution of the given differential equation.

Q15. The population of a village increases continuously at the rate proportional to the number of its inhabitants present at any time. If the population of the village was 20,000 in 1999 and 25,000 in the year 2004, what will be the population of the village in 2009?

A.15.

Let the population at any instant (t) be y.

It is given that the rate of increase of population is proportional to the number of inhabitants at any instant.

$$\therefore \frac{dy}{dt}\alpha y$$

$$\Rightarrow \frac{dy}{dt}=ky \quad \text{(k is constant)}$$

$$\Rightarrow \frac{dy}{y}=kdt$$

Integration both sides, we get:

$$\log y=kt+C..........(1)$$

In the year $1999, t=0 \;\&\; y=20000.$

Therefore, we get:

$$\log 20000=C..........(2)$$

In the year $2004, t=5 \;\&\; y=25000.$

Therefore, we get:

$$\Rightarrow \log 25000=5k+\log 20000$$

$$\Rightarrow 5k=\log\left(\frac{25000}{20000}\right)=\log\left(\frac{5}{4}\right)$$

$$\Rightarrow k=\frac{1}{5}\log\left(\frac{5}{4}\right)..........(3)$$

In the year $2009, t=10 years$

Now, on substituting the values of t, k, and C in equation (1), we get:

$\log y = 10 \times \frac{1}{5} \log\left(\frac{5}{4}\right) + \log(20000)$

$\Rightarrow \log y = \log\left[20000 \times \left(\frac{5}{4}\right)^2\right]$

$\Rightarrow y = 20000 \times \frac{5}{4} \times \frac{5}{4}$

$\Rightarrow y = 31250$

Hence, the population of the village in 2009 will be 31250.

Choose the correct answer:

Q16. The general solution of the differential equation $\frac{ydx - xdy}{y} = 0$ is:

$(A) xy = c$

$(B) x = cy^2$

$(C) y = cx$

$(D) y = cx^2$

A.16.

The given differential equation is:

$\frac{ydx - xdy}{y} = 0$

$\Rightarrow \frac{ydx - xdy}{xy} = 0$

$\Rightarrow \frac{1}{x} dx - \frac{1}{y} dy = 0$

Integration both sides, we get:

$\log|x| - \log|y| = \log k$

$\Rightarrow \log\left|\frac{x}{y}\right| = \log k$

$\Rightarrow \frac{x}{y} = k$

$\Rightarrow y = \frac{1}{k} x$

$\Rightarrow y = Cx, where, C = \frac{1}{k}$

Therefore, option (C) is correct.

Q17. The general equation of a differential equation of the type $\frac{dx}{dy}+P_1x=Q, is:$

$(A) ye^{\int P_1 dy}=\int\left(Q_1e^{\int P_1 dy}\right)dy+C$

$(B) y.e^{\int P_1 dx}=\int\left(Q_1e^{\int P_1 dx}\right)dx+C$

$(C) xe^{\int P_1 dy}=\int\left(Q_1e^{\int P_1 dy}\right)dy+C$

$(D) x.e^{\int P_1 dx}=\int\left(Q_1e^{\int P_1 dx}\right)dx+C$

A.17.

The integrating factor of the given differential equation $\frac{dx}{dy}+P_1x=Q_1$

The general solution of the differential equation is given by,

$x(I.F.)=\int(Q\times I.F.)dy+C$

$\Rightarrow x.e^{\int P_1 dy}=\int\left(Q_1e^{\int P_1 dy}\right)dy+C$

Hence, the correct answer is C.

Q18. The general solution of the differential equation $e^x dy+(ye^x+2x)dx=0$ is:

$(A) xe^y+x^2=C$

$(B) xe^y+y^2=C$

$(C) ye^x+x^2=C$

$(D) ye^y+x^2=C$

A.18.

The given differential equation is:

$e^x dy+(ye^x+2x)dx=0$

$\Rightarrow e^x\frac{dy}{dx}+ye^x+2x=0$

$\Rightarrow \frac{dy}{dx}+y=-2xe^{-x}$

This is a linear differential equation of the form

$\frac{dy}{dx}+Py=Q, where P=1 \& Q=-2xe^{-x}$

$Now, I.F.=e^{\int Pdx}=e^{\int dx}=e^x$

The general solution of the given differential equation is given by,

$$y(I.F.) = \int \left(Q \times I.F.\right) dx + C$$
$$\Rightarrow ye^{x} = \int (-2xe^{-x}.e^{x})dx + C$$
$$\Rightarrow ye^{x} = -\int 2xdx + C$$
$$\Rightarrow ye^{x} = -x^{2} + C$$
$$\Rightarrow ye^{x} + x^{2} = C$$

Therefore, option (c) is correct.