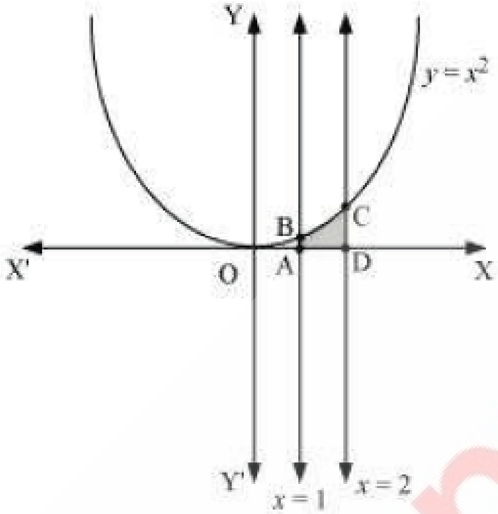
## Miscellaneous

**1.** Find the area under the given curves and given lines:

(i) $y = x^2$, $x = 1$, $x = 2$ and $x$-axis

(ii) $y = x^4$, $x = 1$, $x = 5$ and $x$-axis

i. The required area is represented by the shaded area ADCBA as




$$\text{Area ADCBA} = \int_1^2 y\,dx$$

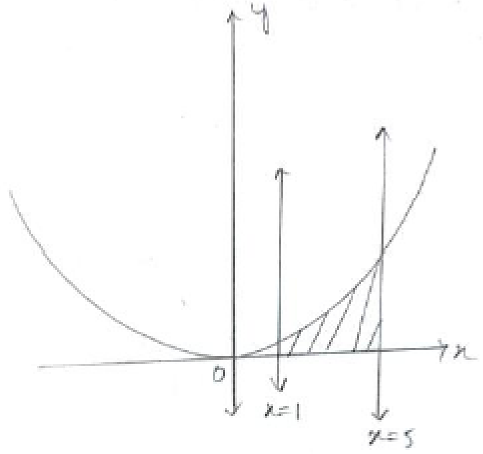
$$= \int_1^2 x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{8}{3} - \frac{1}{3}$$

$$= \frac{7}{3} \text{ units}$$

**ii.)**




$$=\int_1^5 y\,dx$$

$$=\int_1^5 x^4\,dx$$

$$=\left[\frac{x^5}{5}\right]_1^5=\left(\frac{5^5-1^5}{5}\right)$$

$$=\frac{(3125-1)}{5}$$

$$=\frac{3124}{5}=624.8unit^2$$

**2.** Sketch the graph of $y=|x+3|$ and evaluate $\int_{-6}^{0}|x+3|\,dx$.

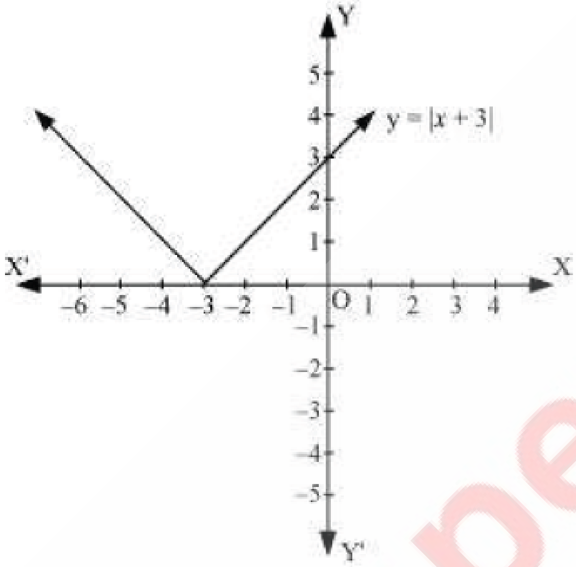
**Answer**

The given equation is $y = |x+3|$

The corresponding values of *x* and *y* are given in the following table.

| *x* | – 6 | – 5 | – 4 | – 3 | – 2 | – 1 | 0 |
|---|---|---|---|---|---|---|---|
| *y* | 3 | 2 | 1 | 0 | 1 | 2 | 3 |

On plotting these points, we obtain the graph of $y = |x+3|$ as follows.




It is known that, $(x+3) \le 0$ for $-6 \le x \le -3$ and $(x+3) \ge 0$ for $-3 \le x \le 0$

$$\therefore \int_{-6}^{0} |(x+3)| dx = -\int_{-6}^{-3} (x+3) dx + \int_{-3}^{0} (x+3) dx$$

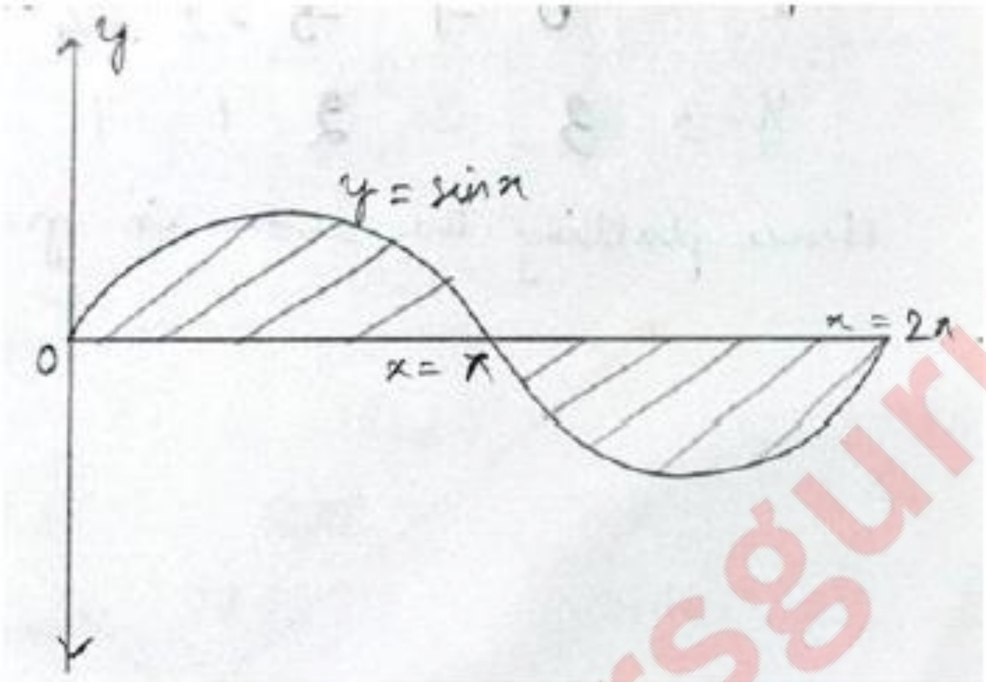
$$= -\left[\frac{x^2}{2} + 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$$

$$= -\left[\left(\frac{(-3)^2}{2} + 3(-3)\right) - \left(\frac{(-6)^2}{2} + 3(-6)\right)\right] + \left[0 - \left(\frac{(-3)^2}{2} + 3(-3)\right)\right]$$

$$= -\left[-\frac{9}{2}\right] - \left[-\frac{9}{2}\right]$$

$$= 9$$

**3.** Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.




$\therefore$ The required area bounded by the curve

$$= \int_0^{\pi} y dx + \int_{\pi}^{2\pi} y dx$$

$$= \int_0^{\pi} \sin x dx + \int_{\pi}^{2\pi} \sin x dx$$

$$= \left| \left[ -\cos x \right]_0^{\pi} \right| + \left| \left[ -\cos x \right]_0^{\pi} \right|$$

$$= \left| -\left[ \cos x - \cos \theta \right] \right| + \left| -\left[ \cos 2\pi - \cos \pi \right] \right|$$

$$= \left| -\left[ -1 - 1 \right] \right| + \left| -\left[ 1 + 1 \right] \right|$$

$$= |2| + |-2| = 2 + 2 = 4 unit^2$$

**4.** Area bounded by the curve $y = x^3$, the $x$-axis and the ordinates $x = -2$ and $x = 1$ is

(A) $-9$ (B) $\frac{-15}{4}$ (C) $\frac{15}{4}$ (D) $\frac{17}{4}$

Answer

For $y=x^3$

| x | 0 | 1 | -1 | 2 | -1 | -2 |
|---|---|---|---|---|---|---|
| y | 0 | 1 | -1 | 8 | -1 | -8 |

$$area(OAB) = \int_0^1 y\,dx$$

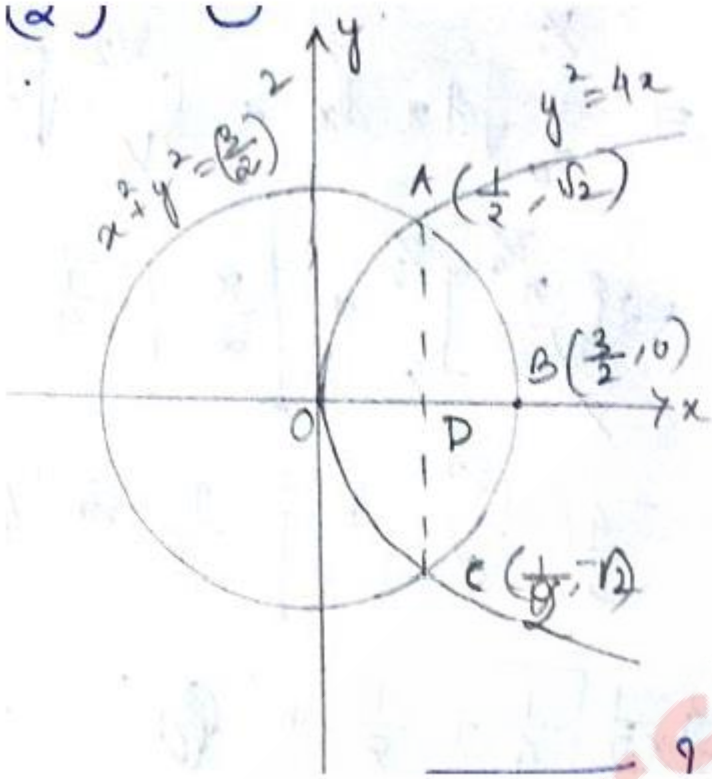
$$= \int_0^1 x^3 dx$$

$$= \left[\frac{x^4}{4}\right]_0^1 = \frac{1}{4}$$

$$area(ODC) = \int_{-2}^0 y\,dx$$

$$= \int_{-2}^0 x^3 dx$$

$$= \left|\left[\frac{x^4}{4}\right]_{-2}^0\right| = \left|\left[\frac{0^4}{4} - \frac{(-2)^4}{4}\right]\right| = 4$$




$\therefore$ Total area of the bounded region $= \frac{1}{4} + 4$

**= 17/4 units**

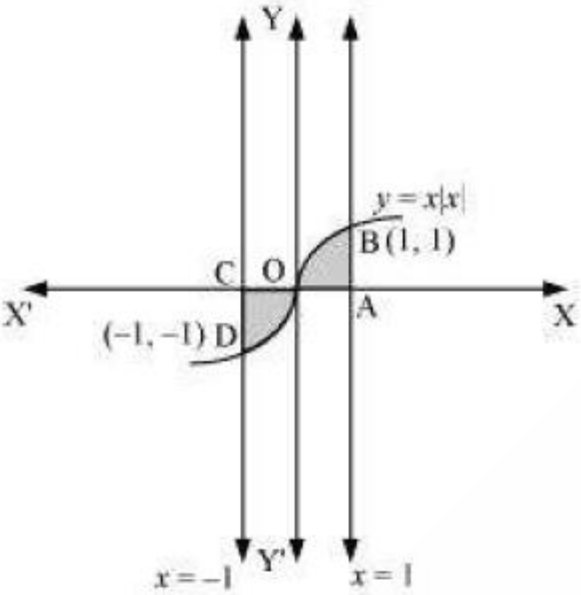
**So, option D is correct**

**5.** The area bounded by the curve $y = x\,|x|$, $x$-axis and the ordinates $x = -1$ and $x = 1$ is given by

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$

[Hint : $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$].

Answer




$$\text{Required area} = \int_{-1}^{1} y\,dx$$

$$= \int_{-1}^{1} x|x|\,dx$$

$$= \int_{-1}^{0} x^2\,dx + \int_{0}^{1} x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1}$$

$$= -\left(-\frac{1}{3}\right) + \frac{1}{3}$$

$$= \frac{2}{3}\text{ units}$$

Thus, option C is correct.