Miscellaneous

Q1. $\left(3x^2-9x+5\right)^9$

A.1. Let $y=\left(3x^2-9x+5\right)^9$

So, $\frac{dy}{dx}=9\left(3x^2-9x+5\right)^8\frac{d}{dx}\left(3x^2-9x+5\right)$

$=9\left(3x^2-9x+5\right)^8\times(6x-9)$

$=27\left(3x^2-9x+5\right)^8(2x-3)$

Q2. $\sin^3 x+\cos^6 x$

A.2. Let $y=\sin^3 x+\cos^6 x$

So, $\frac{dy}{dx}=\frac{d}{dx}\left(\sin^3 x+\cos^6 x\right)$

$=3\sin^2 x\frac{d}{dx}\sin x+6\cos^5 x\frac{d}{dx}\cos x$

$=3\sin^2 x\cos x-6\cos^5 x\sin x$

$=3\sin x\cos x\left(\sin x-2\cos^4\right)$

Q3. $(5x)^{3\cos 2x}$

A.3. Let $y=(5x)^{3\cos 2x}$

Taking log,

$$\log y = 3\cos 2x[\log 5x]$$

Differentiating w r t. x,

$$\frac{1}{y}\frac{dy}{dx} = 3\cos 2x\frac{d}{dx}\log 5x + 3\log 5x\frac{d}{dx}\cos 2x$$

$$= 3\left[\cos 2x\cdot\frac{1}{5x}\frac{d}{dx}(5x) - \log 5x\times\sin 2x\frac{d}{dx}2x\right]$$

$$\frac{dy}{dx} = 3y\left[\cos 2x\times\frac{5}{5x} - \log 5x\cdot\sin 2x\cdot 2\right]$$

$$= 3(5x)^{3\cos 2x}\left[\frac{\cos 2x}{x} - 2\sin 2x\log 5x\right]$$

Q4. $\sin^{-1}\left(x\sqrt{x}\right), 0 \le x \le 1$

A.4. Let $y = \sin^{-1}\left(x\sqrt{x}\right) = \sin^{-1}\left(x^{3/2}\right)$

So, $\dfrac{dy}{dx} = \dfrac{d}{dx}\sin^{-1}\left(x^{3/2}\right)$

$$= \frac{1}{\sqrt{1-\left(x^{3/2}\right)^2}}\frac{d}{dx}\left(x^{3/2}\right)$$

$$= \frac{1}{\sqrt{1-x^3}}\frac{3}{2}x^{3/2-1}$$

$$= \frac{3\sqrt{x}}{2\sqrt{1-x^3}}$$

Q5. $\dfrac{\cos^{-1}\frac{x}{2}}{\sqrt{2x+7}}, -2 < x < 2$

A.5. Let $y = \dfrac{\cos^{-1} x/2}{\sqrt{2x+7}}$

So, $$\frac{dy}{dx}=\frac{\sqrt{2x+7}\frac{d}{dx}\cos^{-1}\frac{x}{2}-\cos^{-1}\frac{x}{2}\frac{d}{dx}\sqrt{2x+7}}{\left(\sqrt{2x+7}\right)^2}$$

$$=\frac{\sqrt{2x+7}\left[\frac{-1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\right]\frac{d}{dx}\left(\frac{x}{2}\right)-\cos^{-1}\frac{x}{2}\times\frac{1}{2\sqrt{2x+7}}\frac{d}{dx}(2x+7)}{2x+7}$$

$$=\frac{-\sqrt{2x+7}\left[\frac{2}{\sqrt{4-x^2}}\right]\times\frac{1}{2}-\cos^{-1}\frac{x}{2}\times\frac{1}{2\sqrt{2x+7}}\times 2}{2x+7}$$

$$=\frac{-\left(\sqrt{2x+7}\right)^2-\cos^{-1}\frac{x}{2}\sqrt{4-x^2}}{\sqrt{4-x^2}\,(2x+7)^{1+1/2}}$$

$$=-\left[\frac{2a+7+\sqrt{4-x^2}\cos^{-1}\frac{x}{2}}{\sqrt{4-x^2}\,(2x+7)^{3/2}}\right]$$

Q6. $\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right], 0<x<\frac{\pi}{2}$

A.6. Let $y=\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$

$$y=\cot^{-1}\left[\frac{\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}-\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}}{\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}-\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}}\right]$$

Putting $1=\cos^2 x/2+\sin^2 x/2$

and $\sin x=2\sin x/2\cos x/2$

$$\Rightarrow y = \cot^{-1}\left[\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right]$$

$$= \cot^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}}\right]$$

$$= \cot^{-1}\left[\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right]$$

$$= \cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow y = \frac{x}{2}$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{2}\right) = \frac{1}{2}$$

Q7. $(\log x)^{\log x}, x > 1$

A.7. Let $y = (\log x)^{\log x}$

Taking log,

$$\log y = \log x \log(\log x)$$

Differentiating w r t. x, we get,

$$\frac{1}{y}\frac{dy}{dx} = \log x \frac{d}{dx}\log(\log x) + \log(\log x)\frac{d}{dx}(\log x)$$

$$= \log x \times \frac{1}{\log x}\frac{d}{dx}\log x + \frac{\log(\log x)}{x}$$

$$= \frac{1}{x} + \frac{\log(\log x)}{x}$$

$$\frac{dy}{dx} = \frac{y}{x}[1 + \log(\log x)].$$

$$= (\log x)^{\log x}\left[\frac{1 + \log(\log x)}{x}\right]$$

Q8. $\cos\,(a\cos x + b\sin x)$, for some constant a and b.

A.8. Let $y = \cos(a\cos x + b\sin x)$

So, $\frac{dy}{dx} = -\sin(a\cos x + b\sin x)\frac{d}{dx}(a\cos x + b\sin x)$

$$= -\sin(a\cos x + b\sin x)[-a\sin x + b\cos x].$$

$$= \sin(a\cos x + b\sin x)(a\sin x - b\cos x)$$

Q9. $(\sin\ x - \cos\ x)^{(\sin x - \cos x)}, \frac{\pi}{4} < x < \frac{3\pi}{4}$

A.9. Let $y = (\sin x - \cos x)^{\sin x - \cos x}$

Taking log,

$$\log y = (\sin x - \cos x)\log(\sin x - \cos x).$$

Differentiating w r t 'x' we get,

$$\frac{1}{y}\frac{dy}{dx} = (\sin x - \cos x)\frac{d\log}{dx}(\sin x - \cos x) + \log(\sin x - \cos x)\frac{d}{dx}(\sin x - \cos x).$$

$$= (\sin x - \cos x) \times \frac{1}{(\sin x - \cos x)} \times (\cos x + \sin x) + \log(\sin x - \cos x)\left(\cos x + \sin x\right)$$

$$= (\cos x + \sin x)[1 + \log(\sin x - \cos x)]$$

$$\frac{dy}{dx} = y(\cos x + \sin x)[1 + \log(\sin x - \cos x)]$$

$$\frac{dy}{dx} = (\sin x - \cos x)^{\sin x - \cos x} \times (\cos x + \sin x)[1 + \log(\sin x - \cos x)]$$

Q10. $x^x + x^a + a^x + a^a$, for some fixed $a > 0$ and b.

A.10. Let $y = x^x + x^a + a^x + a^a$.

So, $\frac{dy}{dx} = \frac{dx^x}{dx} + \frac{dx^a}{dx} + \frac{da^x}{dx} + \frac{da^a}{dx}$.

$\rightarrow \frac{dy}{dx} = \frac{dy}{dx} + ax^{a-1} + a^x \log a + 0.$__________(1)

Where $u = x^x$

$\log u = x \log x$. (Taking log)

$\rightarrow \frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}\log x + \log x\frac{dx}{dx}$ (Differentiation w r t 'x')

$\rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{x}{x} + \log x$

$\rightarrow \frac{dy}{dx} = u[1 + \log x]$

$= x^x[1 + \log x]$

Hence eq^n (1) becomes,

$$\frac{dy}{dx} = x^x[1 + \log x] + ax^{a-1} + a^x \log a.$$

Q11. $x^{x^2-3} + (x-3)^{x^2}$, for $x > 3$

A.11. Let $y = x^{x^2-3} + (x-3)^{x^2}$.

Putting $u = x^{x^2-3}$ and $v = (x-3)^{x^2}$ we get,

$y = u + v$

$$\rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \_\_\_\_\_\_\_\_(1)$$

Now, $u = x^{x^2-3}$

Taking log,

$\log u = \left(x^2 - 3\right)\log x.$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \left(x^2-3\right)\frac{d}{dx}\log x + \log x \frac{d}{dx}\left(x^2-3\right)$$

$$\Rightarrow \frac{du}{dx} = u\left[\frac{x^2-3}{x} + \log x(2x)\right]$$

$$\frac{du}{dx} = x^{x^2-3}\left[\frac{x^2-3}{x} + 2x\log x\right]$$

And $v = (x-3)x^2$

$log\ v = x^2 \log(x-3)$

$$\rightarrow \frac{1}{v}\frac{dv}{dx} = x^2\frac{d}{dx}\log(x-3) + \log(x-3)\frac{d}{dx}x^2$$

$$= \frac{x^2 \times 1 \times}{x-3}\frac{d}{dx}(x-3) + \log(x-3)\cdot 2x$$

$$\frac{dv}{dx} = v\left[\frac{x^2}{x-3} + 2x\log(x-3)\right]$$

$$= (x-3)^{x^2}\left[\frac{x^2}{x-3} + 2x\log(x-3)\right]$$

Hence eq^n (1) becomes

$$\frac{dy}{dx}=x^{\left(x^2-3\right)}\left[\frac{x^2-3}{x}+2x\log x\right]+(x-3)^{x^2}\left[\frac{x^2}{x-3}+2x\log(x-3)\right]$$

Q12. Find $\frac{dy}{dx}$, if $y=12(1-\cos t),\ x=10(t-\sin t),-\frac{\pi}{2}<t<\frac{\pi}{2}$

A.12. Given, $y=12(1-\cos t)$. and $x=10(t-\sin t)$.

Differentiating w r t 't' we get,

$$\frac{dy}{dt}=12\frac{d}{dt}(1-\cos t)=12(0-(-\sin t))=12\sin t.$$

$$\frac{dx}{dt}=10\frac{d}{dt}(t-\sin t)=10(1-\cos t).$$

$$\therefore \frac{dy}{dx}=\frac{dy/dt}{dx/dt}$$

$$=\frac{12\sin t}{10(1-\cos t)}=\frac{12\sin t}{10[1-\cos t]}$$

$$=\frac{12\times 2\sin t/t\cos t/2}{10\times 2\sin^2 t/2}$$

$$\left\{\begin{matrix}\because \sin 2\theta=2\sin\theta\cos\theta \\ \cos 2\theta=1-2\sin^2\theta\end{matrix}\right\}\quad =\frac{6}{5}\frac{\cos t/2}{\sin t/2}=\frac{6}{5}\cot t/2$$

Q13. Find $\frac{dy}{dx}$, if $y=\sin^{-1}x+\sin^{-1}\sqrt{1-x^2}, 0<x<1$

A.13. Given $y=\sin^{-1}x+\sin^{-1}\sqrt{1-x^2}$

$$\frac{dy}{dx}=\frac{d}{dx}\left(\sin^{-1}x+\sin^{-1}\sqrt{1-x^2}\right).$$

$$=\frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1-\left(\sqrt{1-x^2}\right)^2}}\times\frac{d}{dx}\sqrt{1-x^2}$$

$$=\frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1-\left(1-x^2\right)}}\cdot\times\frac{1}{2\sqrt{1-x^2}}\cdot\frac{d}{dx}\left(1-x^2\right).$$

$$=\frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1-1+x^2}}\left\{\frac{-2x}{2\sqrt{1-x^2}}\right\}.$$

$$=\frac{1}{\sqrt{1-x^2}}+\frac{1}{x}\left\{\frac{-x}{\sqrt{1-x^2}}\right\}$$

$$=\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x^2}}$$

= 0.

Q14. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, for,-1 < $x<1$, prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$

A.14. Given, $x\sqrt{1+y}+y\sqrt{1+x}=0.$

$\rightarrow x\sqrt{1+y}=-y\sqrt{1+x}$

Squaring both sides,

$x^2(1+y)=y^2(1+x).$

$\Rightarrow x^2+x^2y=y^2+y^2x$

$\rightarrow x^2-y^2=-x^2y+y^2x.$

$\rightarrow (x-y)(x+y)=-xy(x-y)$

$\Rightarrow x+y=-xy$

$\Rightarrow y+xy=-x$

$\Rightarrow y(1+x)=-x$

$\rightarrow y=\frac{-x}{1+x}.$

$$\therefore \frac{dy}{dx} = -\left\{ \frac{(1+x)\frac{dx}{dx} - x\frac{d}{dx}(1+x)}{(1+x)^2} \right\}$$

$$= \frac{-[1+x-x]}{(1+x)^2} = \frac{-1}{(1+x)^2}.$$

Q15. If $(x-a)^2 + (y-b)^2 = c^2$, for some c > 0, prove that

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

is a constant independent of *a* and *b*.

A.15. Given, $(x-a)^2 + (y-b)^2 = c^2$.

Differentiating w r t '*x*' we get

$$\frac{d}{dx}(x-a)^2 + \frac{d}{dx}(y-b)^2 = \frac{d}{dx}c^2$$

$$\Rightarrow 2(x-a) + 2(y-b)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2(x-a)}{2(y-b)} = -\frac{(x-a)}{(y-b)}$$

Again, $$\frac{d^2y}{dx^2} = -\left\{ \frac{(y-b)\frac{d}{dx}(x-a) - (x-a)\frac{d}{dx}(y-b)}{(y-b)^2} \right\}$$

$$= -\left\{ \frac{(y-b) - (x-a)\frac{dy}{dx}}{(y-b)^2} \right\}$$

$$= -\left\{\frac{(y-b)+(x-a)\dfrac{(x-a)}{(y-b)}}{(y-b)^2}\right\}$$

$$= -\left\{\frac{(y-b)^2+(x-a)^2}{(y-b)^3}\right\}$$

$$= -\frac{c^2}{(y-b)^3} \quad \left\{\because (x-a)^2+(y-b)^2=c^2\right\}$$

Then, L.H.S = $\dfrac{\left\{1+\left(\dfrac{dy}{dx}\right)^2\right\}^{3/2}}{\dfrac{d^2y}{dx^2}} = \dfrac{\left\{1+\dfrac{(x-a)^2}{(y-b)^2}\right\}^{3/2}}{\dfrac{-c^2}{(y-b)^2}}$

$$= \frac{\left\{(y-b)^2+(x-a)^2\right\}^{3/2}}{(y-b)^3} \times \frac{(y-b)^3}{-c^2}$$

$= \dfrac{c^{2\times 3/2}}{-c^2} = \dfrac{c^3}{-c^2} = -c$ Where c is a constant and is independent of a and b.

Q16. If $\cos y = x\cos(a+y)$, with $\cos a \neq 1$, prove that $\dfrac{dy}{dx} = \dfrac{\cos^2(a+y)}{\sin a}$

A.16. Given, $\cos y = x\cos(a+y)$.

$$x = \frac{\cos y}{\cos(a+y)}$$

Differentiating w r t 'y' we get,

$$\frac{dx}{dy} = \frac{d}{dy}\left(\frac{\cos y}{\cos(a+y)}\right).$$

$$= \frac{\cos(a+y)\dfrac{d}{dy}\cos y - \cos y\dfrac{d}{dy}\cos(a+y)}{\cos^2(a+y).}$$

$$= \frac{\cos(a+y)(-\sin y) - \cos y(-\sin(a+y))}{\cos^2(a+y).}$$

$$= \frac{-\cos(a+y)\sin y + \sin(a+y)\cos y}{\cos^2(a+y)}$$

$$= \frac{\sin(a+y)\cos y - \cos(a+y)\sin y.}{\cos^2(a+y)}$$

$$\frac{dx}{dy} = \frac{\sin(a+y-y)}{\cos^2(a+y)} \quad \left\{\because \sin(A-B) = \sin A\cos B - \cos A\sin B\right\}$$

So, $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

Q17. If $x = a\,(\cos t + t\,\sin t)$ and $y = a\,(\sin t - t\,\cos t)$, find $\frac{d^2y}{dx^2}$.

A.17. Given, $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$.

Differentiating w r t. 't' we get,

$$\frac{dx}{dt} = a\frac{d}{dt}(\cos t + t\sin t).$$

$$= a\left(-\sin t + t\frac{d}{dt}\sin t + \sin t\frac{dt}{dt}\right).$$

$$= a(-\sin t + t\cos t + \sin t) = at\cos t$$

$$\frac{dy}{dt} = \frac{ad}{dt}(csint - t\cos t)$$

$$= a\left(\cos t - t\frac{d}{dt}\cos t - \cot\frac{dt}{dt}\right)$$

$$= a(\cos t + t\sin t - \cos t) = atsint$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/at} = \frac{at\sin t}{atcost} = \tan t$$

So, $\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}(\tan t) = \dfrac{d}{dt}(\tan t) \cdot \dfrac{dt}{dx}.$

$= \sec^2 t \times \dfrac{dt}{dx}.$

$= \sec^2 t \times \dfrac{1}{(dx/dt)}$

$= \sec^2 t \times \dfrac{1}{at\ \cos t}$

$= \dfrac{\sec^3 t}{at}$

Q18. If $f(x) = |x|^3$, show that $f''(x)$ exists for all real x and find it.

A.18. Given, $f(x) = |x|^3 = \begin{cases} x^3 & if\ x \geq 0 \\ -x^3 & if\ x < 0 \end{cases}$

For $x \geq 0$, $f(x) = |x|^3 = x^3$

and $f'(x) = 3x^2$
$f''(x) = 6x$

For $x < 0$, $f(x) = |x|^3 = (-x)^3 = -x^3.$

so, $f'(x) = -3x^2$
$f''(x) = -6x$

Hence, $f''(x) = \begin{cases} 6x, & if\ x \geq 0 \\ -6x, & if\ x < 0 \end{cases}$

Q19. Using mathematical induction prove that $\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$ for all positive integers n.

A.19. Given, $\sin(A+B) = \sin A \cos B + \cos A \sin B$

Differentiating w r t. '*x*' we get,

$$\frac{d}{dx}\sin(A+B)=\frac{d}{dx}(\sin A\cos B+\cos A\sin B)$$

$$\to \cos(A+B)-\frac{d}{dx}(A+B)=\sin A\frac{d}{dx}\cos B+\cos B\frac{d}{dx}\sin A+\cos A\frac{d}{dx}\sin B+\sin B\frac{d}{dx}\cos A$$

$$\to \cos(A+B)\left(\frac{dA}{dx}+\frac{dB}{dx}\right)=-\sin A\sin B\frac{dB}{dx}+\cos B\cos A\frac{dA}{dx}+\cos A\cos B\frac{dB}{dx}-\sin A\sin B\frac{dA}{dx}$$

$$\to \cos(A+B)\left(\frac{dA}{dx}+\frac{dB}{dt}\right)=\cos A\cos B\left(\frac{dA}{dx}+\frac{dB}{dx}\right)-\sin A\sin B\left(\frac{dA}{dx}+\frac{dB}{dx}\right)$$

$$=(\cos A\cos B-\sin A\sin B)\left(\frac{dA}{dx}+\frac{dB}{dx}\right).$$

$$\to \cos(A+B)=\cos A\cos B-\sin A\sin B.$$

Q20. Using the fact that sin (A + B) = sin A cos B + cos A sin B and the differentiation, obtain the sum formula for cosines.

A.20. Yes, Let us take $f(x)=|x-1|+|x-2|$.

So, x = 1, x= 2 divides the real line into three disjoint intervals $(-\infty,1],[1,2]$ and $[2,\infty)$.

For $x\in(-\infty,1]$.

$$f(x)=-(x-1)+[-(x-2)]=-x+1-x+2=3-2x.$$

For $x\in[1,2]$.

$$f(x)=(x-1)-(x-2)=1.$$

For $x\in[2,\infty)$

$$f(x)=x-1+x-2=2x-3.$$

Hence, these polynomial fun are all continous and desirable. for all real values of x or, except x = 1 and x = 2.

ie, $\forall x\in R-\{1,2\}$.

For differentiavity at $x = 1$,

LHD = $= \lim_{x \to 1^-} \frac{f(x) - f(1)}{x-1} = \lim_{x \to 1^-} \frac{3-2x-1}{x-1} = \lim_{x \to 1^-} \frac{2-2x}{x-1}$.

$= \lim_{x \to 1^-} \frac{-2(x-1)}{x-1}$

$= \lim_{x \to 1^-} -2$

$= -2$

RHD = $= \lim_{x \to 1^+} \frac{f(x) - f(1)}{x-1} = \lim_{x \to 1^+} \frac{1-1}{x-1} = \lim_{x \to 1^+} \frac{0}{x-1} = 0$.

as L.HD $\neq$ R.HD

f is not differentiable at $x = 1$.

For continuity at $x = 1$.

L.HL= $= \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} = 1$.

RHL = $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 1 = 1$ $\quad \therefore$ LHL = RHS

f is continuous at $x = 1$

For continuity & differentiability at $x = 2$

LHL $= \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 1 = 1$.

R.HS= $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (2x-3) = 4-3 = 1$.

$\because$ LHL = RHL

f is continuous at $x = 2$

LHD $= \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2} = \lim_{x \to 2^-} \frac{1-1}{x-2} = \lim_{x \to 2^-} = \frac{0}{x-2}$
$= 0$

RHD $= \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2} = \lim_{x \to 2^+} \frac{2x-3-1}{x-2}$

$= \lim_{x \to 2^+} \frac{2(x-2)}{x-2}$

$= \lim_{x \to 2^+} 2$

$= 2$

$\because$ LHD $\neq$ RHD

f is not differentiable at $x = 2$.

Q21. Does there exist a function which is continuous everywhere but not differentiable at exactly two points? Justify your answer.

Q22. If $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$, prove that $\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$

A.22. Given, $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$

$= f(x)\begin{vmatrix} m & n \\ b & c \end{vmatrix} - g(x)\begin{vmatrix} l & n \\ a & c \end{vmatrix} + h(x)\begin{vmatrix} l & m \\ a & b \end{vmatrix}.$

$\rightarrow y = f(x)(mc - nb) - g(x)(lc - na) + h(x)(lb - ma)$

Differentiating w r t. 'x' we get,

$\Rightarrow y' = f'(x)(mc - nb) - g'(x)(lc - na) + h'(x)(lb - ma)$

$= \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$

Q23. If $y = e^{a\cos^{-1}x}, -1 \le x \le 1$, show that $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0.$

A.23. Given, $y = e^{a\cos^{-1}x}$

So, $\frac{dy}{dx} = e^{a\cos^{-1}x}\frac{d}{dx}\left(a\cos^{-1}x\right)$

$= e^{a\cdot\cos^{-1}x} \cdot \frac{a \times (-1)}{\sqrt{1-x^2}}$

$\frac{dy}{dx} = -\frac{ae^{a\cos^{-1}x}}{\sqrt{1-x^2}}.$

$\Rightarrow \sqrt{1-x^2}\frac{dy}{dx} = -ae^{a\cos^{-1}x} = -ay.$

Differentiating w r t. 'x' again,

$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{d}{dx}\sqrt{1-x^2} = -a\frac{dy}{dx}.$

$\Rightarrow \sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{1}{2\sqrt{1-x^2}}\frac{dy}{dx}\cdot\frac{d\left(1-x^2\right)}{dx} = -a\frac{dy}{dx}.$

$\Rightarrow \sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{1}{2\sqrt{1-x^2}}\cdot\frac{dy}{dx}\cdot(-2x) = -a\frac{dy}{dx}$

$\rightarrow \left(1+x^2\right)\frac{d^2y}{dx^2}\cdot -x\frac{dy}{dx} = -a\left[\sqrt{1-x^2}\frac{dy}{dx}\right]$

$\rightarrow \left(1+x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -a(-ay).$

$\rightarrow \left(1+x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0.$