Miscellaneous.

Q1. Let A = $\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$, show that $(aI + bA)^n = a^nI + na^{n-1}bA$, where I is the identity matrix of order 2 and $n \in$ N.

A.1.

. We shall prove the result by using principal of mathematical induction
we have,

P (n) :- If A = $\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$, then $(a\,I + b\,A)^n = a^n\,I + na^{n-1}b\,A$ where I is identity matrix of order 2, $n \in N$

P (1): $(a\,I + b\,A)^1 = a^1I + 1 \times a^{1-1}b\,A$
$= a\,I + a^0bA$
$= a^1I + b\,A \;\{\because x^0 = 1\}$

So the result is true for $n = 1$.
Let the result be true for $n = k$. So,
P (k): $(a\,I + b\,A)^u = a^uI + u.\,a^{u-1}b\,A$. _____ (1)
Now, we prove that the result holds for $n = k + 1$,
P (k + 1): $(a\,I + b\,A)^{k+1} = (a\,I + b\,A).\,(a\,I + b\,A)^k$
$= (a\,I + b\,A)\,(a^uI + k\,a^{u-1}b\,A)$ {using eq^n (1)}
$= a.a^kI^2 + k.\,a\,a^{u-1}b\,IA + a^kb.AI + a\,zz^{k\,1}b^2k\,A^2$
$= a^{k+1}I^2 + k\,a^{u-1+1}b\,IA + a^k\,b\,AI + a^{k-1}b^2\,k\,A^2$. _____ (2)

Now, $I^2 = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I$

$IA = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} = A$

$AI = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} = A.$

$A^2 = A.A = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} = 0.$

Putting these values in eq^n (2). We get,
P (k + 1) $= a^{k+1}\,I + kab\,A + a^{k+1-1}b\,A + 0.$
$= a^{k+1}\,I + (k + 1)\,a^{(k+1)-1}b\,A.$

$\therefore$ The result is true for $n = k + 1$. Thus by principle of mathematical induction

$(a\,I + b\,A)^n = a^n I + na^{n-1}bA$ for A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

holds true for all natural number *n*.

Q2. If A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$, n ∈ N.

A.2. We have,

(E) P (*n*) : If A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. then $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ $n \in$ N.

P (1) : $A^1 = \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix} = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

So, the result holds true for *n* = 1.
Let the result be for *n* = *k*. So,

P (*k*): $A^u = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$

Then P (*k*+1): $A^{k+1} = A^k . A = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \\ 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \\ 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \end{bmatrix}$

$= A^{k+1} = \begin{bmatrix} 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \end{bmatrix}$

∴ The result holds for *n* = *k* + 1. Hence,

$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ holds for all natural number.

Q3. If A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, where n is any positive integer.
A.3. We have,

(E) P (n) : A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then A^n = $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

P (1): $A^1 = \begin{bmatrix} 1+2.1 & -4.1 \\ 1 & 1-2.1 \end{bmatrix} = \begin{bmatrix} 1+2 & -4 \\ 1 & 1-2 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

So, the result is true for $n = 1$.
Let the result be true for $n = k$.

P (k) = $A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$

So, p (k + 1) $A^{K+1} = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 3(1+2k)+(-4k)\times 1 & -4(1+2k)+(-4k)(-1) \\ 3k+1\times(1-2k) & -4\times k+(1-2k)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k+2k-1 \end{bmatrix} = \begin{bmatrix} 3+2k & -4-4k \\ 1+k & -1-2k \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+2k & -4(1+k) \\ 1+k & 1-2-2x \end{bmatrix}$$

$$= \begin{bmatrix} 1+2(1+k) & -4(1+k) \\ 1+k & 1-2(1+k) \end{bmatrix}$$

$\therefore$ The results also holds for $n = k + 1$. Hence, $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

Holds for all natural number n.

Q4. If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix.

A.4. Given, A and B are symmetric matrices.

(E) Then A' = A and B' = B.

Now, (AB – BA)' = (AB)' – (BA)'
= B'A' – A'B'
= BA – AB
= – (AB – BA).

Hence, AB – BA is skew-symmetric matrix

Q5. Show that the matrix B'AB is symmetric or skew symmetric according as A is symmetric or skew symmetric.

A.5. We have,

(E) (B'AB)' = [B'(AB]'
= (AB)'(B')'
= B'A'B.

When A is symmetric, A' = A
(B'AB)' = B'AB
ie, B'AB is symmetric.

And when A is skew-symmetric, $A^1 = -A$
(B'AB)' = – B'AB.
ie, B'AB is skew-symmetric.

Q6. Find the values of x, y, z if the matrix A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ **satisfy the equation A'A = I.**

A.6. Given, $A^1 = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$. Then, $A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

Since, $A'A = I$ we can write,

$$\Rightarrow \begin{bmatrix} 2 & 2y & z \\ x & y & z \\ x & -y & z \end{bmatrix}\begin{bmatrix} 0 & x & z \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-2x+2x & 2yz-yz-yz & z^2+z^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equating the corresponding elements of the matrices,

$2x^2 = 1$, $6y^2 = 1$, $3z^2 = 1$

$\Rightarrow x^2 = \frac{1}{2}$ $\quad \Rightarrow y^2 = \frac{1}{6}$ $\quad \Rightarrow z^2 = \frac{1}{3}$

$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$ $\quad \Rightarrow y = \pm\frac{1}{\sqrt{6}}$ $\quad \Rightarrow z = \pm\frac{1}{\sqrt{3}}$

Q7. For what values of x : $[1 \quad 2 \quad 1]\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix}$ **= 0?**

A.7. $[1 \quad 2 \quad 1]\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0.$

$$\Rightarrow [1+4+1 \quad 2+0+0 \quad 0+2+2]\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0.$$

$$\Rightarrow [6 \quad 2 \quad 4]\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$[6 \times 0 + 2 \times 2 + 4 \times x] = 0.$

$[0 + 4 + 4x] = 0$

Q8. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$.

A.8. Given A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.

$$\text{So; } A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3\times3+1\times(-1) & 3\times1+1\times2 \\ -1\times3+2\times(-1) & -1\times1+2\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\therefore A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 6 & 6 \end{bmatrix} = 0.$$

Hence Showed.

Q9. Find x, if $[x - 5 - 1]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

A.9. Given $[x - 5 - 1]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

(k) $\Rightarrow [x - 0 - 2 \quad 0 - 10 - 0 \quad 2x - 5 - 3]\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$\Rightarrow [x - 2 \quad -10 \quad 2x - 8]\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$\Rightarrow (x - 2)x - 40 + (2x - 8) = 0$

$\Rightarrow x^2 - 2x - 40 + 2x - 8 = 0$

$\Rightarrow x^2 = 48$

$\Rightarrow x = \pm\sqrt{48}$

$\Rightarrow x = \pm 4\sqrt{3}$

Q10. A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below:

Market **Products**

I	**10,000**	**2,000**	**18,000**
II	**6,000**	**20,000**	**8,000**

(a) If unit sale prices of *x, y and z* are ₹ 2.50, ₹ 1.50 and ₹ 1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are ₹ 2.00, ₹ 1.00 and 50 paise respectively. Find the gross profit.

A.10. Let $A = \begin{bmatrix} x & y & z \\ 10,000 & 2,000 & 18,000 \\ 6,000 & 20,000 & 8,000 \end{bmatrix} \begin{matrix} \\ \text{Market I} \\ \text{Market II} \end{matrix}$

(*a*) The unit sale price matrix, $B = \begin{bmatrix} 2\cdot 5 \\ 1\cdot 5 \\ 1 \end{bmatrix}$

$\therefore$ Total revenue matrix AB, $= \begin{bmatrix} 10,000 & 2,000 & 18,000 \\ 6,000 & 20,000 & 8,000 \end{bmatrix} \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 25,000 + 3000 + 18,000 \\ 15,000 + 30,000 + 8,000 \end{bmatrix}$

$= \begin{bmatrix} 46,000 \\ 53,000 \end{bmatrix}$ Revenue from market I   Revenue from market II

Hence, total revenue in Market I & II are ₹ 46,000 and ₹ 53,000 respectively.

(*a*) The unit costs price matrix, $B = \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$ {50 paisa = 0.5}

$\therefore$ Total cost matrix AB = $\begin{bmatrix} 10,000 & 2,000 & 18,000 \\ 6,000 & 20,000 & 8,000 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$

$= \begin{bmatrix} 20,000 + 2,000 + 9,000 \\ 12,000 + 20,000 + 4,000 \end{bmatrix}$

$= \begin{bmatrix} 31,000 \\ 36,000 \end{bmatrix}$

Hence, Profit matrix = Revenue matrix – Cost matrix

$= \begin{bmatrix} 46,000 \\ 53,000 \end{bmatrix} - \begin{bmatrix} 31,000 \\ 36,000 \end{bmatrix} = \begin{bmatrix} 15,000 \\ 17,000 \end{bmatrix}$

$\therefore$ Gross profit = ₹ 15,000 + ₹ 17,000 = ₹ 32, 000.

Q11. Find the matrix X so that $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

A.11. Let $m \times n$ be the order of matrix X (M) Then $X_{m\times n} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}_{2\times 3}$

By matrix multiplication rule,

and $n = 2$
$m = 2$

So, X is a 2×2 order matrix, let it be X $= \begin{bmatrix} w & x \\ y & z \end{bmatrix}$.

We have, $\begin{bmatrix} w & x \\ y & z \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.

$\Rightarrow \begin{bmatrix} w+4x & 2w+5x & 3w+6x \\ y+4z & 2y+5z & 3y+6z \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.

Equating the corresponding elements of the matrices,

$w + 4x = -7$______ (i)
$2w + 5x = -8$______ (ii)
$3w + 6x = -9$______ (iii)
$y + 4z = 2$______ (iv)
$2y + 5z = 4$______ (v)
$3y + 6z = 6$______ (vi)

Multiplying eq^n (i) by (2) and subtracting from (ii) we get,
$2w + 5x - 2(w + 4x) = -8 - 2 \times (-7)$
$\Rightarrow 2w + 5x - 2w - 8x = -8 + 14$
$\Rightarrow -3x = 6$
$\Rightarrow x = \dfrac{6}{-3} \Rightarrow x = -2$

Putting $x = -2$ in eq^n (i) we get
$\Rightarrow$ w + 4(–2) = – 7
$\Rightarrow$ w – 8 = – 7
$\Rightarrow$ w = 8 – 7
$\Rightarrow$ w = 1

Multiplying eq^n (w) by 2 and subtracting if from (v) we get
$2y + 5z - 2(y + 4z) = 4 - 2 \times 2$
$\Rightarrow 2y + 5z - 2y - 8z = 0$
$\Rightarrow -3z = 0$
$\Rightarrow z = 0$

Putting $z = 0$ in eq^n (iv) we get,
$y + 4 \times 0 = 2$
$\Rightarrow y = 2$

Hence, the matrix X is $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

Q12. If A and B are square matrices of the same order such that AB = BA, then prove by induction that $AB^n = B^nA$. Further, prove that $(AB)^n = A^nB^n$ for all n $\in$ N.

A.12. We have, AB = BA. (given)

(E) P (n):AB’ = B’A.
P (i):$AB^1 = B^1A$. $\Rightarrow$ AB = BA
so, the result is true for $n = 1$.
Let the result be true for $n = k$.
P (k):$AB^k = B^kA$
Then,
P (k + 1) : $AB^{k+1} = A.B^k.B = B^kA.B = B^k.BA$ $\{\because AB = BA\}$
$= B^{k+1}.A$.
So, $AB^{k+1} = B^{k+1}A$.
$\therefore$ The result also holds for $n = k + 1$.
Hence, $AB^n = B^nA$ holds for all natural number ‘n’.

Choose the correct answer in the following questions:

Q13. If A = $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that A^2 = I, then

(A) $1 + \alpha^2 + \beta\gamma = 0$ **(B) $1 - \alpha^2 + \beta\gamma = 0$**
(C) $1 - \alpha^2 - \beta\gamma = 0$ **(D) $1 + \alpha^2 - \beta\gamma = 0$**

A.13. Given, A = $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

And $A^2 = I$.

$$\Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2+\beta\gamma & \alpha\beta-\beta\alpha \\ \alpha\gamma-\alpha\gamma & \gamma\beta+\alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \alpha^2+\beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating the corresponding elements of the matrices we get,

$$\alpha^2 + \beta\gamma = 1.$$

$$\Rightarrow 1 - \alpha^2 - \beta\gamma = 0.$$

$\therefore$ Option (c) is correct.

Q14.If the matrix A is both symmetric and skew symmetric, then

(A) A is a diagonal matrix **(B) A is a zero matrix**
(C) A is a square matrix **(D) None of these**

A14. Given, A is both symmetric and skew-symmetric.

(E) Then, A' = A ____ (1) and A' = – A ____ (2)

So using (2), A' = – A.

A = – A {eq^n (I)}

$\Rightarrow$ A + A = 0

$\Rightarrow$ 2A = 0

$\Rightarrow$ A = 0.

$\therefore$ A is a zero matrix

So, option B is correct.

Q15. If A is square such that A^2 = A, then $(I + A)^3 - 7A$ is equal to

(A) A **(B) I – A** **(C) I** **(D) 3A**

A.15. Given, $A^2 = A$.

(E) we need to calculate,

$(I + A)^3 - 7A = I^3 + A^3 + 3IA\,(I + A) - 7A$ $\{(x + y)^3 = x^3 + y^3 + 3xy\,(a + y)\}$

$= I + A^2.\,A + 3I^2A. + 3IA^2 - 7A.$ $\left\{\begin{array}{l} \because I^3 = I,\ I^2 = I. \\ IA = AI = A \\ \text{property of identity matrix} \\ \text{and } A^2 = A \text{ given.} \end{array}\right.$

$= I + A.A. + 3IA + 3AI.\ 7A.$

$= I + A.A. + 3IA + 3IA - 7A$

$= I + 7A - 7A$

$= I.$

So, option (c) is correct.