Ex.3.3

Q1. Find the transpose of each of the following matrices:

(i) $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ **(iii)** $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

A.1. (i) Let $A = \begin{bmatrix} 5 \\ 1/2 \\ -1 \end{bmatrix}_{[3\times1]}$

Then, A' = [5 ½ − 1] $_{1\times3}$

(ii) Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}_{2\times2}$

Then, $A' = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}_{2\times2}$.

(iii) Let $A = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

Then $A' = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

Q2. **If** $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$**, then verify that**

(i) (A + B)′□= A′□+ B′, **(ii) (A – B) ′□= A′□– B′**

A.2. Given, $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

Then, $A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

$$A + B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -1-4 & 2+1 & 3-5 \\ 5+1 & 7+2 & 9+0 \\ -2+1 & 1+3 & 1+1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$$

L.H.S. = (A + B)' = $\begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$

R.H.S = A'+ B' = $\begin{bmatrix} -1 & 5 & -2 \\ 1 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1-4 & 5+1 & -2+1 \\ 2+1 & 7+2 & 1+3 \\ 3-5 & 9+0 & 1+1 \end{bmatrix} = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$ = L H S.

$\therefore$ (A + B)' = A'+ B'

(ii) A – B = $\begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -1-(-4) & 2-1 & 3-(5) \\ 5-1 & 7-2 & 9-0 \\ -2-1 & 1-3 & 1-1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$

L.H.S. = (A – B)' = $\begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$

R.H.S. = A' – B' = $\begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1-(-4) & 5-1 & -2-1 \\ 2-1 & 7-2 & 1-3 \\ 3-(-5) & 9-0 & 1-1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$

$\therefore$ (A – B)' = A' – B'

Q3. **If** $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$**, then verify that**

(i) (A + B) '= A'+ B' **(ii) (A – B) '= A'– B'**

A.3. Given, A' = $\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and B' = $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$.

Then (A')' = A = $\begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ and B' = $\begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

(i) A + B = $\begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 3-1 & -1+2 & 0+1 \\ 4+1 & 2+2 & 1+3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$

$$\text{L·H·S·} = (A + B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

$$\text{R·H·S} = A' + B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3-1 & 4+1 \\ -1+2 & 2+2 \\ 0+1 & 1+3 \end{bmatrix} = \text{L H S.}$$

$$\because (A + B)' = A' + B'$$

$$\text{(ii) } A - B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 3-(-1) & -1-2 & 0-1 \\ 4-1 & 2-2 & 1-3 \end{bmatrix} = \begin{bmatrix} 1 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$$

$$\text{L·H·S·} = (A - B)' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

$$\text{R.H.S.} = A' - B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3-(-1) & 4-1 \\ -1-2 & 2-2 \\ 0-1 & 1-3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix} = \text{L.H.S.}$$

$$\because (A - B)' = A' - B'$$

Q4. **If** $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ **and** $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$**, then find (A + 2B) ′**

A.4. Given, $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

Then, $B' = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$

$\therefore$ (A + 2B)’ = (A + 2B)’ = A’ + 2B’

$$\begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} + 2 \cdot \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 2 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-2 & 3+2 \\ 1+0 & 2+4 \end{bmatrix} = \begin{bmatrix} -4 & 5. \\ 1 & 6 \end{bmatrix}$$

Q5. **For the matrices A and B, verify that (AB)′□= B′A′, where**

(i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$ **(ii)** $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

A.5. (i) Given, A= $\begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

Then, A’ = $\begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$ and B’ = $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}_{3\times1} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}_{1\times3} = \begin{bmatrix} 1\times(-1) & 1\times2 & 1\times1 \\ -4\times(-1) & -4\times2 & -4\times1 \\ 3\times-1 & 3\times2 & 3\times1 \end{bmatrix}_{3\times3}$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

$$= L\cdot H\cdot S\cdot = (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}.$$

$$R\cdot H\cdot S\cdot = B'\ A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}_{3\times1} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}_{1\times3} = \begin{bmatrix} -1\times1 & -1\times(-4) & -1\times3 \\ 2\times1 & 2\times(-4) & 2\times3 \\ 1\times1 & 1\times(-4) & 1\times3 \end{bmatrix}_{3\times3}$$

$$= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix} = R\cdot H\cdot S$$

$\because$ (AB)’ = B’ A’.

Given,

(ii) 1 A = $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ and B = [1 5 7]

Then A' = [0 1 2] and B' = $\begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$.

(i) AB = $\underset{3\times1}{\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}} \underset{1\times3}{\begin{bmatrix} 1 & 5 & 7 \end{bmatrix}} = \underset{1\times3}{\begin{bmatrix} 0\times1 & 0\times5 & 0\times7 \\ 1\times1 & 1\times5 & 1\times7 \\ 2\times1 & 2\times5 & 2\times7 \end{bmatrix}} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$

L.H.S. = (AB)' = $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$

R.H.S. = b' A' = $\begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 12 \end{bmatrix} = \begin{bmatrix} 1\times0 & | \times1 & 1\times2 \\ 5\times0 & 5\times1 & 5\times2 \\ 7\times0 & 7\times1 & 7\times2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$ L.H.S.

∴ (AB)' = B' A'.

Q6. **If (i)** $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$**, then verify that A′ A = I**

(ii) If $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$**, then verify that A′ A = I**

A.6. (i) Given, A = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

Then, A' = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

∴A' A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cot\alpha \end{bmatrix}$

= $\begin{bmatrix} \cos\alpha\cdot\cos\alpha + (-\sin\alpha)(-\sin\alpha) & \cos\alpha\cdot\sin\alpha + (-\sin\alpha)\cos\alpha \\ \sin\alpha\cdot\cos\alpha + \cos\alpha(-\sin\alpha) & \sin\alpha\cdot\sin\alpha + \cos\alpha\cos\alpha \end{bmatrix}$

$$= \begin{bmatrix} \cos^2 a + \sin^2 a & \cos a \sin a - \sin a \cos a \\ \sin a \cos a - \cos a \sin a & \sin^2 a + \cos^2 a \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \{\because \cos^2 x + \sin^2 x = 2\}$$

= A ' A = 1.

Given,

(ii) 1 A = $\begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cot\alpha & \sin\alpha \end{bmatrix}$

Then, A ' = $\begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cot\alpha & \sin\alpha \end{bmatrix}$

∴A' A = $\begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cot\alpha & \sin\alpha \end{bmatrix}\begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

$$= \begin{bmatrix} \sin\cdot\alpha\sin\alpha + (\cos\alpha)(-\cos\alpha) & \sin\alpha\cdot\cos\alpha + (-\cos\alpha)\cos\alpha \\ \cos\alpha\cdot\sin\alpha + \sin\alpha(-\cos\alpha) & \cos\alpha\cdot\cos\alpha + \sin\alpha\sin\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2\alpha + \cos^2\alpha & \sin\alpha\cot\alpha - \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha - \sin\alpha\cos\alpha & \cos^2\alpha + \sin^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \{\because \cos^2 x + \cos\cos^2 x = 1\}.$$

$\Rightarrow$ A' A. = I

Q7. (i) Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ **is a symmetric matrix.**

(ii) Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ **is a skew symmetric matrix.**

A.7. (i) Given A = $\begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$.

Then, A' = $\begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$

∴A' = A.

Here, A is *symmetric* matrix

(i) Given, A = $\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

Then, A' = $\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} = (-1)\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

$\Rightarrow$ A' = (–1) A.

$\Rightarrow$ A' = – A.

Hers A is *a* show *symmetric* matrix.

Q8. For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ **verify that**

(i) (A + A′) is a symmetric matrix
(ii) (A – A′) is a skew symmetric matrix

A.8. Given, A = $\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

Then, A' = $\begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

(i) Let P = A + A' = $\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1+1 & 5+6 \\ 6+5 & 7+7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

So, P' = $\begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$ = P

i e, (A + A')' = A + A'.

Hence, A + A' is *symmetric* matrix.

(ii) Let Q = A – A' = $\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1-1 & 5-6 \\ 6-5 & 7-7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

So, $Q^1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = (-1)\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = (-1)\,Q.$

$\Rightarrow Q^1 = -Q.$

i e, (A – A')' = – (A – A').

Have, A – A' is *a* show *symmetric* matrix

Q9. Find $\frac{1}{2}(A + A')$ **and** $\frac{1}{2}(A - A')$**, when** $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

A.9. Given, $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

Then, $A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$

So, $A + A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & a-a & b-b \\ -a+a & 0 & c-c \\ -b+b & -c+c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$

And $A - A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & a-(-a) & b-(-b) \\ -a-a & 0 & c-(-c) \\ -b-b & -c-c & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix}$$

$\therefore \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 2a & 2b \\ -2a & 0a & 2c \\ -2b & -2c & 0 \end{bmatrix} = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}.$

Q10. Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

(i) $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ **(iv)** $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

Choose the correct answer in the Exercises 11 and 12.

A.10. (i) Let A = $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$.

Then, A' = $\begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$.

Let P = $\frac{1}{2}$ (A + A') = $\frac{1}{2}\left\{\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}+\begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}\right\}=\frac{1}{2}\begin{bmatrix} 3+3 & 5+1 \\ 1+5 & -1+ \end{bmatrix}$

$=\frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}=\begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$.

Then, P' = $\begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$ = P.

$\therefore$ P = $\frac{1}{2}$ (A + A') is *symmetric* matrix

Let Q = $\frac{1}{2}$ (A + A') = $\frac{1}{2}\left\{\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}-\begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}\right\}=\frac{1}{2}\begin{bmatrix} 3-3 & 5-1 \\ 1-5 & -1-(-1) \end{bmatrix}$

$=\frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$.

Then Q.' = $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$ = (–1) $\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ = (–1) Q.

$\Rightarrow$ Q.' = – Q,

$\therefore$ Q = $\frac{1}{2}$ (A – A') is *a symmetric* matrix

Now, P + Q = $\frac{1}{2}$ (A + A') + $\frac{1}{2}$ (A – A')

$$\Rightarrow P + Q = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 3+0 & 3+2 \\ 3-2 & -1+0 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = A.$$

This A is represented as a sun of *symmetric* and skew *symmetric* matrix

(iii) Let $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$

Then $A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$

Now, $A + A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 6+6 & -2+(-2) & 2+2 \\ -2+(-2) & 3+3 & -1+(-1) \\ 2+2 & -1+(-1) & 3+3 \end{bmatrix} = \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$$

Let $P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Then, $P' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = P'$

$\therefore P = \frac{1}{2}(A + A')$ is *a*syntri matrix.

$$A - A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Let $Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma & 0 \\ \sigma & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$

$$Q' = (-1)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -Q.$$

$\Rightarrow$ Q' = – Q.

$\therefore Q = \frac{1}{2}$ (A – A') is *a* skew *symmetric* matrix

$$\text{Have } P + Q = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = A.$$

Then A is represented as *a* sum of symmetric & skew symmetric matrix

$$\text{(iii) Let } A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$\text{Then, } A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(A + A') = \frac{1}{2}\left\{\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 3+3 & 3-2 & -1-4 \\ -2+3 & -2-2 & 1-5 \\ -4-1 & -5+1 & 2+2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix}$$

$$\text{Then } P' = \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} = P.$$

$\therefore P = \frac{1}{2}$ (A + A') is *symmetric* matrix.

Let Q = $\frac{1}{2}$ (A – A’) = $\frac{1}{2}\left\{\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & 45 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right\}$

$$= \frac{1}{2}\begin{bmatrix} 3-3 & 3-(-2) & -1-(-4) \\ -2-3 & -2-(-2) & 1-(-5) \\ -4-(-1) & -5-1 & 2-2 \end{bmatrix}.$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 5/2 & 3/2 \\ -5/2 & 0 & 3 \\ -3/2 & -3 & 0 \end{bmatrix}$$

Then Q’ = $\begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix} = (-1)\begin{bmatrix} 0 & 5/2 & 3/2 \\ 5/2 & 0 & 3 \\ 3/2 & 3 & 0 \end{bmatrix}$ = (–1) Q.

$\Rightarrow$ Q’ = – Q.

∴ Q = $\frac{1}{2}$ (A – A’) is *a* skew *symmetric* matrix

∴ P + Q = *symmetric*

= .

.

$$= \begin{bmatrix} 3 & 6/2 & -2/2 \\ :/2 & -2 & 1 \\ -/2 & -5 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = A.$$

Thus A is represented as *a* sum of *symmetric* and skew symmetric matrix.

(iv) Let A = $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

Then A’ = $\begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$.

Let P = $\frac{1}{2}$ (A + A') = $\frac{1}{2}\left\{\begin{bmatrix}1 & 5\\-1 & 2\end{bmatrix}+\begin{bmatrix}1 & -1\\5 & 2\end{bmatrix}\right\}=\frac{1}{2}\begin{bmatrix}1+1 & 5-1\\-1+5 & 2+2\end{bmatrix}$

$=\frac{1}{2}\begin{bmatrix}2 & 4\\4 & 4\end{bmatrix}=\begin{bmatrix}1 & 2\\2 & 2\end{bmatrix}$.

Then, P' = $\begin{bmatrix}1 & 2\\2 & 2\end{bmatrix}$ = P.

∴ P = $\frac{1}{2}$ (A +A ') is symmetric matrix

Let Q = $\frac{1}{2}$ (A – A') = $\frac{1}{2}\left\{\begin{bmatrix}1 & 5\\-1 & 2\end{bmatrix}-\begin{bmatrix}1 & -1\\5 & 2\end{bmatrix}\right\}=\frac{1}{2}\begin{bmatrix}1 & -1 & 5-(-1)\\-1 & -5 & 2-2\end{bmatrix}$

$=\frac{1}{2}\begin{bmatrix}0 & 6\\-6 & 0\end{bmatrix}=\begin{bmatrix}0 & 3\\-3 & 0\end{bmatrix}$.

Thus, Q' = $\begin{bmatrix}0 & -3\\3 & 0\end{bmatrix}$ = (-1) $\begin{bmatrix}0 & 3\\-3 & 0\end{bmatrix}$ = (–1) Q.

Þ Q' = – Q.

∴ Q = $\frac{1}{2}$ (A – A') is *a* skew symmetric matrix

∴ P + Q = $\begin{bmatrix}1 & 2\\2 & 2\end{bmatrix}+\begin{bmatrix}0 & 3\\-3 & 0\end{bmatrix}=\begin{bmatrix}1+0 & 2+3\\2-3 & 2+0\end{bmatrix}$

$=\begin{bmatrix}1 & 5\\-1 & 2\end{bmatrix}$ = A.

Thus A is represented as *a* sum of *symmetric* and skew *symmetric* matrix

Q11. If A, B are symmetric matrices of same order, then AB – BA is a
(A) Skew symmetric matrix **(B) Symmetric matrix**
(C) Zero matrix **(D) Identity matrix**

A.11. Given A and B are *symmetric* matrices,

(E) Then, A' = A and B' = B.

Now, (AB – BA)' = (AB)' – (BA)'

= B’A’ – A’B’.

= BA – AB

(AB – BA)’ = – (AB – BA)

AB – BA is *a* skew *symmetric* matrix

∴ Option A is correct.

Q12. **If** $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ **and A + A′□= I, then the value of ∝□is**

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{3}$

(C) π **(D)** $\frac{3\pi}{2}$

A.12. Given, A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$. Then, A’ = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

and A + A’ = I.

Þ $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Þ $\begin{bmatrix} 2\cos\alpha + \cos\alpha & -\sin\alpha + \sin\alpha \\ \sin\alpha - \sin\alpha & \cos\alpha + \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Þ $\begin{bmatrix} 2\cot\alpha & 0 \\ 0 & 2\cot\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Equating the corresponding element of the matrix we get,

$2\cos\alpha = 1$

Þ $\cos\alpha = \frac{1}{2}$

Þ $\alpha = \cos - \frac{11}{2} = \cos^{-1}\left(\cos\frac{\pi}{3}\right)$

Þ $\alpha = \frac{\pi}{3}$.

Option B is correct