Ex.3.2

Q1 **Let** $A=\begin{bmatrix}2&4\\3&2\end{bmatrix}, B=\begin{bmatrix}1&3\\-2&5\end{bmatrix}, C=\begin{bmatrix}-2&5\\3&4\end{bmatrix}$

Find each of the following:

(i) A + B **(ii) A – B** **(iii) 3A – C** **(iv) AB (v) BA**

A.1. $A=\begin{bmatrix}2&4\\3&2\end{bmatrix}, B=\begin{bmatrix}1&3\\-2&5\end{bmatrix},\ C=\begin{bmatrix}-2&5\\3&4\end{bmatrix}$

(i) A + B = $\begin{bmatrix}2&4\\3&2\end{bmatrix}+\begin{bmatrix}1&3\\-2&5\end{bmatrix}=\begin{bmatrix}2+1&4+3\\3-2&2+5\end{bmatrix}=\begin{bmatrix}3&7\\1&7\end{bmatrix}$

(ii) A–B = $\begin{bmatrix}2&4\\3&2\end{bmatrix}-\begin{bmatrix}1&3\\-2&5\end{bmatrix}=\begin{bmatrix}2-1&4-3\\3-(-2)&2-5\end{bmatrix}=\begin{bmatrix}3&1\\5&-3\end{bmatrix}.$

(iii) 3A–C = $3\begin{bmatrix}2&4\\3&2\end{bmatrix}-\begin{bmatrix}-2&5\\3&4\end{bmatrix}=\begin{bmatrix}3\times2&3\times4\\3\times3&3\times2\end{bmatrix}-\begin{bmatrix}-2&5\\3&4\end{bmatrix}=\begin{bmatrix}6&12\\9&6\end{bmatrix}\begin{bmatrix}-2&5\\3&4\end{bmatrix}$

$=\begin{bmatrix}6-(-2)&12-5\\9-3&6-4\end{bmatrix}=\begin{bmatrix}8&7\\6&2\end{bmatrix}$

(iv) AB = $\begin{bmatrix}2&4\\3&2\end{bmatrix}\begin{bmatrix}1&3\\-2&5\end{bmatrix}=\begin{bmatrix}2\times1+4\times(-2)&2\times3+4\times5\\3\times1+2\times(-2)&3\times3+2\times5\end{bmatrix}=\begin{bmatrix}2-8&6+20\\3-4&9+10\end{bmatrix}$

$=\begin{bmatrix}-6&26\\-1&19\end{bmatrix}$

(v) BA = $\begin{bmatrix}1&3\\-2&5\end{bmatrix}\begin{bmatrix}2&4\\3&2\end{bmatrix}=\begin{bmatrix}1\times2+3\times3&1\times4+3\times2\\-2\times2+5\times3&-2\times4+5\times2\end{bmatrix}=\begin{bmatrix}2+9\\-4+15\end{bmatrix}$

$=\begin{bmatrix}11&10\\11&2\end{bmatrix}$

Q2. **Compute the following:**

(i) $\begin{bmatrix}a&b\\-b&a\end{bmatrix}+\begin{bmatrix}a&b\\b&a\end{bmatrix}$ **(ii)** $\begin{bmatrix}a^2+b^2&b^2+c^2\\a^2+c^2&a^2+b^2\end{bmatrix}+\begin{bmatrix}2ab&2bc\\-2ac&-2ab\end{bmatrix}$

(iii) $\begin{bmatrix}-1&4&-6\\8&5&16\\2&8&5\end{bmatrix}+\begin{bmatrix}12&7&6\\8&0&5\\3&2&4\end{bmatrix}$ **(iv)** $\begin{bmatrix}\cos^2x&\sin^2x\\\sin^2x&\cos^2x\end{bmatrix}+\begin{bmatrix}\sin^2x&\cos^2x\\\cos^2x&\sin^2x\end{bmatrix}$

A.2. (I) $\begin{bmatrix}a&b\\-b&a\end{bmatrix}+\begin{bmatrix}a&b\\b&a\end{bmatrix}=\begin{bmatrix}a+a&b+b\\-b+b&a+a\end{bmatrix}=\begin{bmatrix}2a&2b\\0&2a\end{bmatrix}.$

(II) $$\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix} = \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$$

$$= \begin{bmatrix} (a+b)^2 (b+c)^2 \\ (a-c)^2 (a-b)^2 \end{bmatrix} \qquad \begin{Bmatrix} (x+y)^2 = x^2+2xy+y^2 \\ (x-y)^2 = x^2-2xy+y^2 \end{Bmatrix}$$

(III) $$\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix} = \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

(iv) $$\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix} = \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Q3. Compute the indicated products.

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ **(v)** $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

(vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

A.3. (i) $$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}_{2\times} = \begin{bmatrix} a\times a + b\times b & a\times(-b)+b\times a \\ -b\times a + a\times b & (b)\times(b)+a\times a \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & -ab+ab \\ -ab+ab & b^2+a^2 \end{bmatrix}_{3\times3} = \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}_{2\times2.}$$

(ii) $$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times3} [234] = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times1} [234]_{1\times3} = \begin{bmatrix} 1\times2 & 1\times3 & 1\times4 \\ 2\times2 & 2\times3 & 2\times4 \\ 3\times2 & 3\times3 & 3\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}_{3\times3.}$$

(iii) $$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}_{2\times3} = \begin{bmatrix} 1\times1+(-2)\times2 & 1\times2+(-2)\times3 & 1\times3+(-2)\times1 \\ 2\times1+3\times2 & 2\times2+3\times3 & 2\times3+3\times1 \end{bmatrix}_{2\times3}$$

$$= \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix}_{2\times3} = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}_{2\times3}.$$

$$(\text{iv}) \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}_{3\times3} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}_{3\times3}$$

$$= \begin{bmatrix} 2\times1+3\times0+4\times3 & 2\times(-3)+3\times2+4\times0 & 2\times5+3\times4 \\ 3\times1+4\times0+5\times3 & 3\times(-3)+4\times2+5\times0 & 8\times5+4\times4+5\times5 \\ 4\times1+5\times0+6\times0 & 4\times(-3)+5\times2+6\times0. & 4\times5+5\times4+6\times5 \end{bmatrix}_{3\times3}$$

$$= \begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix} = \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 57 \\ 22 & -2 & 70 \end{bmatrix}_{3\times3}$$

$$(\text{v}) \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}_{2\times3} = \begin{bmatrix} 2\times1+1\times(-1) & 2\times0+1\times2 & 2\times1+1\times1 \\ 3\times1+2\times(-1) & 3\times0+2\times2 & 3\times1+2\times1 \\ -1\times1+1\times(-1) & -1\times0+1\times2 & -1\times1+1\times1 \end{bmatrix}_{3\times3}$$

$$= \begin{bmatrix} 2-1 & 0+2 & 2+1 \\ 3-2 & 0+4 & 3+2 \\ -1-1 & 0+2 & -1+1 \end{bmatrix}_{3\times3} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}_{3\times3}$$

$$(vi) \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}_{2\times3} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}_{3\times2} = \begin{bmatrix} 3\times2+(-1)\times1+3\times3 & 3\times(3)+(-1)\times0+3\times1 \\ -1\times2+0\times1+2\times3 & -1\times(-3)+0\times0+2\times1 \end{bmatrix}_{2\times2}$$

$$= \begin{bmatrix} 6-1+9 & -9+0+3 \\ -2+0+6 & 3+0+2 \end{bmatrix}_{2\times2} = \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}_{2\times2}$$

Q4. **if** $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ **and** $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$ **then compute (A+B) and (B – C). Also, verify that A + (B – C) = (A + B) – C.**

A.4 $$A + B = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1+3 & 2-1 & -3+2 \\ 5+4 & 0+2 & 2+5 \\ 1+2 & -1+0 & 1+3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$$

$$B - C = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 3-4 & -1-1 & 2-2 \\ 4-0 & 2-3 & 5-2 \\ 2-1 & 0-(2)\ 3 & 3 \end{bmatrix}$$

$$B-C=\begin{bmatrix}-1 & -2 & 0\\ 4 & -1 & 2\\ 1 & 2 & 0\end{bmatrix}$$

Now

L.H.S. = $a+(b-c)$ = $\begin{bmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{bmatrix}+\begin{bmatrix}-1 & -2 & 0\\ 4 & -1 & 3\\ 1 & 2 & 0\end{bmatrix}=\begin{bmatrix}1-1 & 2-2 & -3+0\\ 5+4 & 0-1 & 2+3\\ 1+1 & -1+2 & 1+0\end{bmatrix}$

$$=\begin{bmatrix}0 & 0 & -3\\ 9 & -1 & 5\\ 2 & 1 & 1\end{bmatrix}$$

R.H.S. = $(a+b)-c$ = $\begin{bmatrix}4 & 1 & -1\\ 9 & 2 & 7\\ 3 & -1 & 4\end{bmatrix}-\begin{bmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{bmatrix}=\begin{bmatrix}4-4 & 1-1 & -1-3\\ 9-0 & 2-3 & 7-2\\ 3-1 & -1-(2) & 4-3\end{bmatrix}$

$$=\begin{bmatrix}0 & 0 & -3\\ 9 & -1 & 5\\ 2 & 1 & 1\end{bmatrix}=\text{L.H.S.}$$

$\therefore A+(B-C)=(A+B)-C$

Q5. **If** $A=\begin{bmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{bmatrix}$ **and** $B=\begin{bmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{bmatrix}$ **then compute 3A – 5B.**

A.5. $3a-5b=3\times\begin{bmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{bmatrix}-5\times\begin{bmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{bmatrix}$

$$=\begin{bmatrix}3\times\frac{2}{3} & 3\times1 & 3\times\frac{5}{3}\\ 3\times\frac{1}{3} & 3\times\frac{2}{3} & 3\times\frac{4}{3}\\ 3\times\frac{7}{3} & 3\times2 & 3\times\frac{2}{3}\end{bmatrix}-\begin{bmatrix}5\times\frac{2}{5} & 5\times\frac{3}{5} & 5\times1\\ 5\times\frac{1}{5} & 5\times\frac{2}{5} & 5\times\frac{4}{5}\\ 5\times\frac{7}{5} & 5\times\frac{6}{5} & 5\times\frac{2}{5}\end{bmatrix}$$

$$=\begin{bmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{bmatrix}-\begin{bmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{bmatrix}=\begin{bmatrix}2-2 & 3-3 & 5-5\\ 1-1 & 2-2 & 4-4\\ 7-7 & 6-6 & 2-2\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 6 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Q6. **Simplify** $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

A.6. $.\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

$$(E)\ =\begin{bmatrix}\cot^2\theta & \cot\theta\sin\theta\\ -\cos\theta\sin\theta & \cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin 2\theta & -\sin\theta\cos\theta.\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta+\sin^2\theta & \cos\theta\sin\theta-\sin\theta\cos\theta\\ -\cos\theta\sin\theta+\sin\theta\cos\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Q7. **Find X and Y, if**

(i) $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ **and** $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

(ii) $2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ **and** $3X+2Y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$

A.7. .(i) $x+y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$

$x-y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

Adding e^n (i) and (ii) we get,

$$x+y+x-y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$$

$$\Rightarrow 2x=\begin{bmatrix}7+3 & 0+0\\ 2+\theta & 5+3\end{bmatrix}=\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}$$

$$\Rightarrow \quad x=\frac{1}{2}\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\times 10 & \frac{1}{2}\times 0\\ \frac{1}{2}\times 2 & \frac{1}{2}\times 8\end{bmatrix}=\begin{bmatrix}5 & 0\\ 1 & 4.\end{bmatrix}$$

Subtracting $eq^n(ii)$ from(i) we get,

$$x+y-(x-y)=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}-\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$$

$$\Rightarrow x+y-x+y=\begin{bmatrix}7-3 & 0-0\\ 2-0 & 5-3\end{bmatrix}$$

$$\Rightarrow 2y=\begin{bmatrix}4 & 0\\ 2 & 2\end{bmatrix}$$

$$\Rightarrow\ Y=\frac{1}{2}\begin{bmatrix}4 & 0\\ 2 & 2\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\times 4 & \frac{1}{2}\times 0\\ \frac{1}{2}\times 2 & \frac{1}{2}\times 2\end{bmatrix}=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$$

(ii) $2x + 3y = \begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ —— (1)

$2x + 2y = \begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$ ——— (2)

Multiplying eq^n by 2 and $q^n(u)$

$2 \times (2x+ 3y–) = 2 \times \begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$

$\Rightarrow\ 4x + 6y = \begin{bmatrix}2\times 2 & 2\times 3\\ 2\times 4 & 2\times 0\end{bmatrix}$

$\Rightarrow\ 4x + 6y = \begin{bmatrix}4 & 6\\ 8 & 0\end{bmatrix} \to (iii).$

$3 \times (3x+ 2y) = 3 \times \begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$

$\Rightarrow\ 9x + 6y = \begin{bmatrix}3\times(2) & 3\times(-2)\\ 3\times(-1) & 3\times 5\end{bmatrix}=\begin{bmatrix}6 & -6\\ -3 & 15\end{bmatrix}\to(iv)$

Subtracting eq^n (iii) from (iv) we get,

$4x+ 6y–(4x+ 6y) = \begin{bmatrix}6 & -6\\ -3 & 15\end{bmatrix}-\begin{bmatrix}4 & 6\\ 8 & 0\end{bmatrix}$

$\Rightarrow 5x = \begin{bmatrix}6-4 & -6-6\\ -3-8 & 15-0\end{bmatrix}=\begin{bmatrix}2 & -12\\ -11 & 15\end{bmatrix}$

$$\Rightarrow\ x=\frac{1}{5}\begin{bmatrix}2 & -12\\ -11 & 15\end{bmatrix}=\begin{bmatrix}2\times\frac{1}{5} & -12\times\frac{1}{5}\\ -11\times\frac{1}{5} & 15\times\frac{1}{5}\end{bmatrix}=\begin{bmatrix}2/5 & -12/5\\ -11/5 & 3\end{bmatrix}$$

Fromeq^n (u);

$$3y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}-2x=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}-\begin{bmatrix}2\times\frac{2}{5} & 2\times\left(-\frac{12}{5}\right)\\ 2\times(-11/5) & 2\times 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}-\begin{bmatrix}4/5 & -24/5\\ -22/5 & 6\end{bmatrix}$$

$$=\begin{bmatrix}2-4/5 & 3-\left(-24/5\right)\\ 4-\left(-22/5\right) & 0-6\end{bmatrix}$$

$$=\begin{bmatrix}\frac{5\times2-4}{5} & \frac{3\times5+24}{5}\\ \frac{4\times5+22}{5} & -6\end{bmatrix}$$

$$=\begin{bmatrix}\frac{10-4}{5} & \frac{15+24}{5}\\ \frac{20+22}{5} & -6\end{bmatrix}$$

$$y=\frac{1}{3}\begin{bmatrix}6/5 & 3q/5\\ 42/5 & -6\end{bmatrix}=\begin{bmatrix}1/3\times\frac{6}{5} & 1/3\times\frac{39}{5}\\ 1/3\times\frac{42}{5} & \frac{1}{3}\times(-6)\end{bmatrix}$$

$$=\begin{bmatrix}2/5 & 13/5\\ 14/5 & -2\end{bmatrix}$$

Q8. **Find X, if** $\mathrm{Y}=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$ **and** $2\mathrm{X}+\mathrm{Y}=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$

A.8. Give, $2x + 4 = \begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$

$$\Rightarrow\quad 2x=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}-y=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}-\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}1-3 & 0-2\\ -3-1 & 2-4\end{bmatrix}$$

$$\Rightarrow\quad x=\frac{1}{2}\begin{bmatrix}-2 & -2\\ -4 & -2\end{bmatrix}=\begin{bmatrix}-2\times\frac{1}{2} & -2\times\frac{1}{2}\\ -4\times\frac{1}{2} & -2\times\frac{1}{2}\end{bmatrix}=\begin{bmatrix}-1 & -1\\ -2 & -1\end{bmatrix}$$

Q9.Find x **and** y**, if** $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$

A.9. Given, $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$

$$\Rightarrow\quad \begin{bmatrix}2\times1 & 2\times3\\ 2\times0 & 2\times x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$$

$$\Rightarrow\quad \begin{bmatrix}2+y & 6+0\\ & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$$

Equating the corresponding elements of the matrices we get,

$2 + y = 5$

$\Rightarrow y = 5 - 2 = 3$

And $2x + 2 = 8$

$\Rightarrow 2x = 8 - 2$

$\Rightarrow \quad x = \dfrac{6}{2}$

$\Rightarrow x = 3$

$\because x = 3, y - 3.$

Q10. Solve the equation for x, y, z and t, if $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

A.10. $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2\times x & 2\times 2 \\ 2\times y & 2\times t \end{bmatrix} + \begin{bmatrix} 3\times 1 & 3\times(-1) \\ 3\times 0 & 3\times 2 \end{bmatrix} = \begin{bmatrix} 3\times 3 & 3\times 5 \\ 3\times 4 & 3\times 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 21 \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}.$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y+0 & 21+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

Equating the corresponding elements of the matrices u get,

$2x + 3 = 9$

$\Rightarrow 2x = 9 - 3$

$\Rightarrow \quad x = \dfrac{6}{2}$

$\Rightarrow \boxed{x = 3}$

$2z - 3 = 15$

$\Rightarrow 2z = 15 + 3$

$\Rightarrow \quad z = \dfrac{18}{2}$

$\Rightarrow \boxed{z = 9}$

$2y = 12$

$\Rightarrow \quad y = \dfrac{12}{2} \Rightarrow \boxed{y = 6}$

$2t + 6 = 18$

$\Rightarrow 2t = 18 - 6$

$\Rightarrow 2t = 12 \Rightarrow t = \dfrac{12}{2} \Rightarrow \boxed{t = 6}$

Q11. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$**,find the values of x and y.**

A.11. Given, $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

(E) $\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2x - y \\ 3x + y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}.$$

Equating the corresponding elements of the matrices we get,

$2x - y = 10$ _______ (i)
$3x + y = 5$ ______ (ii)
Adding (i) and (i) we get,
$2x - y + 3x + y = 10 + 5.$
$\Rightarrow 5x = 15$

$$\Rightarrow x = \frac{15}{5} \Rightarrow \boxed{x = 3}$$

$\Rightarrow 5x = 15$

$$\Rightarrow \quad x = \frac{15}{5} \Rightarrow x = 3$$

Putting $x = 3$ in (i) we gel,
$2 \times 3 - y = 10$
$\Rightarrow 6 - 10 = y$
$\Rightarrow \boxed{y = -4}$

Q12. Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$**, find the values of *x*, *y*, *z* and *w*.**

A.12. Given, $x\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}.$

$$(B) \Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+2+w & 2w+3 \end{bmatrix}$$

Equating the corresponding elements of the matrices we get ,

$$\begin{aligned} &3x = x+4 \\ &3x - x = 4 \\ \Rightarrow\ &2x = 4 \\ \Rightarrow\ &x = \frac{4}{2} \\ \Rightarrow\ &x = 2 \end{aligned}$$

$$\begin{aligned} &3y = 6+x+y \\ \Rightarrow\ &3y - y = 6+2 \\ \Rightarrow\ &2y = 8 \\ \Rightarrow\ &y = 8/2 \\ \Rightarrow\ &y = 4 \end{aligned}$$

$$\begin{aligned} &3w = 2w+3 \\ \Rightarrow\ &3w - 2w = 3 \\ \Rightarrow\ &aw = 3 \end{aligned}$$

$$\begin{aligned} &3z = -1+z+6 \\ &3z - z = -1+3 \\ \Rightarrow\ &2z = 2 \\ \Rightarrow\ &z = 2/2 \\ \Rightarrow\ &z = 1. \end{aligned}$$

Q13. If F $(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$**, show that F(*x*) F(*y*) = F(*x* + *y*).**

A.13. Given, $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}.$

So, F(y) $= \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\therefore f(x)f(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{cases} \cos x cor y + (-\sin x)\sin y + 0 & \cos x(-\sin y) + (-\sin x)\cos y + 0 & \cos x \cdot 0 + (\sin x)\cdot 0 + 1\times 0 \\ \sin x\cos y + \cos x\sin y + 0 & \sin x(-\sin y) + \cos x\cos y + 0 & \sin x\cdot 0 + \cos \\ 0\times\cos y + 0\times\sin y + 1\times 0 & 0\times(-\sin y) + 0\times\cos y + 1\times 0 & 0\times 0 + 0\times 0 + 0 \text{ a} \end{cases}$$

$$= \begin{bmatrix} \cos x\cos y - \sin x\sin y & -[\cos x\sin y + \sin x\cos y] & 0 \\ \sin x\cos y + \cos x\sin y & -\sin x\sin y + \cos x\cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{cases} \because \cos(x+y) = \cos x\cos y - \sin x\sin y. \\ \sin(x+y) = \sin x\sin y + \cos x\cos. \\ = \sin x\cos y + \cos x\sin y. \end{cases}$$

$= \mathrm{F}(x + y)$

Hence, $\mathrm{F}(x) - \mathrm{F}(y) = \mathrm{F}(x + y)$

Q14. Show that

(i) $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

A.14. (i) L.H.S. = $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5\times 2 + (-1)\times 3 & 5\times 1 + (-1)\times 4 \\ 6\times 2 + 7\times 3 & 6\times 1 + 7\times 4 \end{bmatrix}$

$$= \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

R.H.S $= \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 2\times 5 + 1\times 6 & 2\times(-1) + 1\times 7 \\ 3\times 5 + 4\times 6 & 3\times(-1) + 4\times 7 \end{bmatrix}.$

$$= \begin{bmatrix} 10+6 & -2+7. \\ 15+24 & -3+28 \end{bmatrix} = \begin{bmatrix} 16 & 5. \\ 29 & 25 \end{bmatrix}.$$

$\because$ L.H.S $\neq$ R.H.S.

(ii)L.H.S = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 1\times(-1) + 2\times 0 + 3\times 2 & 1\times 1 + 2\times(-1) + 3\times 3 & 1\times 0 + 2\times 1 + 3\times 4 \\ 0\times(-1) + 1\times 0 + 0\times 2 & 1\times 0 + 1\times(-1) + 0\times 3 & 0\times 0 + 1\times 1 + 0\times 4 \\ 1\times(-1) + 1\times 0 + 0\times 2 & 1\times 1 + 1\times(-1) + 0\times 3 & 1\times 0 + 1\times 1 + 0\times 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+6 & 1-2+9 & 2+12 \\ 0 & -1 & 1 \\ -1 & 1-1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

R.H.S = $\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} -1\times1+1\times0+0\times1 & -1\times2+|\times|+0\times1 & -1\times3+1\times0+0\times0 \\ 0\times1+(-1)\times0+| \times1 & 0\times2+(-1)\times1+| \times1 & 0\times3+(-1)\times0+1\times0. \\ 2\times1+3\times0+4\times1 & 2\times2+3\times1+4\times1 & 2\times3+3\times0+4\times0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2+1 & -3 \\ 1 & -1+1 & 0 \\ 2+4 & 4+3+4 & 6 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

$\because$ L.H.S $\neq$ R.H.S.

Q15. **Find** A^2 **– 5A + 6I, if** $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

A.15. .Given, A = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$.

$A^2 = AA^2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 2\times2+0\times2+1\times1 & 2\times0+0\times1+1\times(1) & 2\times1+0\times3+1\times0 \\ 2\times2+1\times2+3\times1 & 2\times0+1\times1+3\times(-1) & 2\times1+1\times3+3\times0 \\ 1\times2+(-1)\times2+0\times1 & 1\times0+(-1)\times1+0\times(-1) & 1\times1+(-1)\times3+0\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & -1 & 2 \\ 4+2+3 & 1-3 & 2+3 \\ 2-2 & -1 & 1-3 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ .9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

So, $a^2 – 5a + 6$](Identify matrix is of order 3 as a is of order 3)

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10+6 & -1-0+0 & 2-5+0 \\ 9-10+0 & -2-5+6 & 5-15+0 \\ 0-5+0 & -1-(-5)+0 & -2-0+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & -2 \end{bmatrix}$$

Q16 **If** $A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$ **prove that** $A^3-6A^2+7A+2I=0$

A.16. given A = $\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$

$$A^2 = AA = \begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+0\times0+2\times2 & 1\times0+2\times0+2\times0 & 1\times2+0\times1+2\times3\\ 0\times1+2\times0+1\times2 & a\times0+2\times2+1\times0 & 0\times2+2\times1+1\times3\\ 2\times1+0\times0+3\times2 & 2\times0+0\times2+3\times0 & 2\times2+0\times1+3\times3\end{bmatrix}$$

$$=\begin{bmatrix}1+4&0&2+6\\2&4&2+3\\2+6&0&4+9\end{bmatrix}=\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}$$

$$A^3 = A^2A = \begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$$

$$=\begin{bmatrix}5\times1+0\times0+8\times2 & 5\times0+0\times2+8\times0 & 5\times2+0\times1+8\times3\\ 2\times1+4\times0+5\times2 & 2\times0+4\times2+5\times0 & 2\times2+4\times1+5\times3\\ 8\times1+0\times0+13\times2 & 8\times0+0\times2+0\times0 & 8\times2+0\times1+13\times3\end{bmatrix}$$

$$\begin{bmatrix}5+16&0&10+24\\2+10&8&4+4+15\\8+26&0&16+39\end{bmatrix}=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}$$

So, $A^3-6A^2+7A+2I$

$$=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}-6\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}+7\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}+2\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}-\begin{bmatrix}30&0&48\\12&24&30\\48&0&78\end{bmatrix}+\begin{bmatrix}7&0&14\\0&14&7\\14&0&21\end{bmatrix}+\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$$

$$=\begin{bmatrix}21-30+7+2 & 0-0+0+0 & 34-48+14+0\\ 12-12+0+0 & 8-24+14+2 & 23-30+7+0\\ 34-48+14+0 & 0-0+0+0 & 55-78+21+2\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=0$$ (2000 matrix)

Hence proud

Q17. **If** $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$ **and** $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, **find *k* so that** $A^2 = kA-2I$

A.17. given, $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$

$A^{2}=A.\ A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$

$=\begin{bmatrix}3\times3+(-2)\times4 & 3\times(-2)+(-2)\times(-2)\\ 4\times3+(-2)\times4 & 4\times(-2)+(-2)\times(-2)\end{bmatrix}$

$=\begin{bmatrix}9-8 & -6+4\\ 12-8 & -8+4\end{bmatrix}=\begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}.$

So, $A^2 = xA-2I$.

$\Rightarrow\begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}=u\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}-2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$

$\Rightarrow\begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}=\begin{bmatrix}3x & -2x\\ 4x & -2x\end{bmatrix}-\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}=\begin{bmatrix}3x-2 & -2x\\ 4x & -2x-2\end{bmatrix}.$

Comparing the corresponding elements of the matrices we get

$1=3x-2 \qquad -2=-2k \qquad 4=4x \qquad -4=-2x-2$

$\Rightarrow 1+2=3x \qquad \Rightarrow a=\frac{-2}{-2} \qquad \Rightarrow x=\frac{4}{4} \qquad 7-2x=-4+2$

$\Rightarrow n=\frac{3}{3} \qquad \Rightarrow x=1 \qquad \Rightarrow u=1 \qquad \Rightarrow x=\frac{-2}{-2}$

$\Rightarrow x=1 \qquad \Rightarrow x=1$

$\boxed{\therefore x=1}$

Q18. **If** $A=\begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}$, **and I is the identity matrix of order 2, show that**

$I+A=(I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

A.18. given, $A=\begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

L.H.S = I + A = $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}=\begin{bmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{bmatrix}$

$$\text{R.H.S} = (I - A)\begin{bmatrix} \cot\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha & 0 \end{bmatrix}\right\}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cot\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan{}^{4}\!/_{2} \\ -\tan\alpha/2 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1\times\cos\alpha + \tan{}^{\alpha}\!/_{2}\sin\alpha & 1\times\in\sin\alpha) + \tan\frac{\alpha}{2}\cdot\cot\alpha \\ -\tan\frac{\alpha}{2}\times\cot\alpha + 1\times\sin\alpha & -\tan\frac{\alpha}{2}(-\sin\alpha) + 1\times\cot\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \frac{\sin{}^{\alpha}\!/_{2}}{\cos{}^{\alpha}\!/_{2}}\times\sin\alpha & -\sin\alpha + \frac{\sin{}^{\alpha}\!/_{2}}{\cos{}^{\alpha}\!/_{2}}\cdot\cos\alpha \\ -\frac{\sin{}^{\alpha}\!/_{2}}{\cos{}^{\alpha}\!/_{2}}\cos\alpha + \sin\alpha & \frac{\sin{}^{\alpha}\!/_{2}}{\cos{}^{\alpha}\!/_{2}}\sin\alpha + \cot\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\cos\alpha\cos{}^{\alpha}\!/_{2} + \sin{}^{\alpha}\!/_{2}\sin\alpha}{\cos\alpha/2} & \frac{-\sin\alpha\cos{}^{\alpha}\!/_{2} + \cos\alpha\sin{}^{\alpha}\!/_{2}}{\cos{}^{\alpha}\!/_{2}} \\ \frac{-\sin{}^{\alpha}\!/_{2}\cos\alpha + \cos{}^{\alpha}\!/_{2}\sin\alpha}{\cos{}^{\alpha}\!/_{2}} & \frac{\sin{}^{\alpha}\!/_{2}\sin\alpha + \cot{}^{\alpha}\!/_{2}\cos\alpha}{\cos{}^{\alpha}\!/_{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\cos\left(\alpha - {}^{\alpha}\!/_{2}\right)}{\cos{}^{\alpha}\!/_{2}} & \frac{-\sin\left(\alpha - {}^{\alpha}\!/_{2}\right)}{\cos{}^{\alpha}\!/_{2}} \\ \frac{\sin\left(\alpha - {}^{\alpha}\!/_{2}\right)}{\cos - {}^{\alpha}\!/_{2}} & \frac{\cos\left(\alpha - {}^{\alpha}\!/_{2}\right)}{\cos{}^{\alpha}\!/_{2}} \end{bmatrix} \left\{\begin{array}{l} \because \cos(x - \alpha y) = \cos x\cos y + \\ \quad \sin x\sin y \\ \sin(x - \alpha) = \sin x\cos y - \cos x \\ \quad \sin y \end{array}\right.$$

$$= \begin{bmatrix} \frac{\cos{}^{\alpha}\!/_{2}}{\cos{}^{\alpha}\!/_{2}} & \frac{-\sin{}^{\alpha}\!/_{2}}{\cos{}^{\alpha}\!/_{2}} \\ \frac{\sin{}^{\alpha}\!/_{2}}{\cos{}^{\alpha}\!/_{2}} & \frac{\cos{}^{\alpha}\!/_{2}}{\cos{}^{\alpha}\!/_{2}} \end{bmatrix} = \begin{bmatrix} 1 - \tan{}^{\alpha}\!/_{2} \\ \tan\frac{\alpha}{2} 1 \end{bmatrix} = \text{L.H.S.}$$

$$\because I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Q19. A trust fund has ₹ 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide ₹ 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of:
(a) ₹1800 (b) ₹2000

A.19. let x be the investment in the first bond out of total ₹ 30,000.
(M) then, investment use for the second bond = ₹ 30,000–x.
Hence, investment matrix A = $[x \quad 30{,}000 - x]_{1\times 2}$
As there is 5% and 7% interest paid the first and second , (per year)

The annual interest matrix B = $\begin{bmatrix} 5\% \\ 7\% \end{bmatrix}_{2\times1} = \begin{bmatrix} \frac{5}{100} \\ 7/100 \end{bmatrix}_{2\times1}$

∴ Annual total interest, AB = $\begin{bmatrix} x & 30,000-x \end{bmatrix}\begin{bmatrix} 5/100 \\ 7/100 \end{bmatrix}$

$= \left[x \times \frac{5}{100} + (30000-x) \times \frac{7}{100} \right]_{1\times1}$

$= \frac{5x + 2,10,000 - 7x}{100} = \frac{2,10,000-2x}{100.}$

⇒Manual total interest = ` $\frac{210,000-2x}{100.}$

(a) Given, manual total interest = ` 1800.

$\Rightarrow ₹\frac{2,10,000-2x}{100} = ₹1800.$

$\Rightarrow ₹2,10,000 - 2x = ₹1.80,000$

⇒` 210.000 –`180,000 = 2x

$\Rightarrow x = ₹\frac{30,000}{2}$

⇒x = ₹ 15, 000

∴investment in first and second bond is x = 15,000 and ₹ 30,000 –x = ₹ 30,000–₹ 15,000–₹ 15,000 respectively.

(B) given ,Annual total interest = ` 2000.

$\Rightarrow \frac{₹2,10,000-2x}{100} = ₹2000.$

⇒₹ 2,10, 000 – ₹ 2,00,000

⇒₹ 2,10, 000 –₹ 2; 000 = 2x

$\Rightarrow x = ₹\frac{10,500}{2}$

⇒x = ₹ 5,000

∴investment in first and second hand is x = ₹ 5,000 and.₹ 30,000–x = 30,000–₹ 5,000 = 25,000 respectively

Q20. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are `80, `60 and`40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Assume X, Y, Z, W and P are matrices of order 2 × *n*, 3 × *k*, 2 × *p*, *n* × 3 and *p* × *k*, respectively. Choose the correct answer in Exercises 21 and 22.

A.20. Number of books matrix A = $\begin{bmatrix} 10 \text{ dozen} & 8 \text{ dozen} & 10 \text{ dozen} \end{bmatrix}$

A = $\begin{bmatrix} 10\times12 & 8\times12 & 10\times12 \end{bmatrix}$ $\{\because 1 \text{ dosen } = 12\}$

$= \begin{bmatrix} 120 & 96 & 120 \end{bmatrix}$

Selling price of each book metric, B = $\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}_{3\times1}$

∴total amount received from selling all the books is

$$AB = \begin{bmatrix} 120 & 96 & 120 \end{bmatrix} \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}_{3\times 1}$$

$= [120 \times 80 + 96 \times 60 + 120 \times 40]_{1\times 1}$

$= (9600 + 5760 + 4800)$

= ` 20160

Order of matrices , $x \to 2 \times n$

$y \to 3 \times x$

$z \to 2 \times p$

$w \to n \times 3$

$p \to p \times x.$

Q21. **The restriction on n, k and p so that PY + WY will be defined are:**

(A) $k = 3, p = n$ **(B) k is arbitrary, $p = 2$**

(C) p is arbitrary, $k = 3$ **(D) $k = 2, p = 3$**

A.21. For, PY + WY to be defined

(E) $P_{p\times k} \cdot Y_{3\times x}$ should be seen that number of columns of p

Should, be equal to no of nous of y i.e,[$x = 3$]

$n \times 3$ and $(PY)_{(p\times x)}$

similarity, in $W^{n\times 3} Y^{3\times x}$ we see thatno of columns of

W = no of rows in Y and (WY) $n \times x$.

Again,PY + WY is defined if order of $(PY)_{p\times k}$ and $(WY)_{n\times u}$ are the sameie ,$\boxed{P = n}$

so, option a is correct .

Q22. **If $n = p$, then the order of the matrix 7X – 5Z is:**

(A) $p \times 2$ **(B) $2 \times n$** **(C) $n \times 3$** **(D) $p \times n$**

A.22. The order of $7x$–5z will be same as order of x and z

as× has order $2 \times n$ and z has order $2 \times$ p.and given n = p.

$7x$–5z has order $2 \times n$

∴option b is correct.