## Miscellaneous

**Find the value of the following:**

**Q1.** $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

**A.1.** $\text{Cos}^{-1}\left(\cos\frac{13\pi}{6.}\right) = \cos^{-1}\left(\cos\frac{12\pi+\pi}{6}\right)$

$= \cos^{-1}\left(\cos\frac{12\pi}{6}+\frac{\pi}{6}\right)$

$= \cos^{-1}\left[\cos^{-1}\left(2\pi+\frac{\pi}{6}\right)\right]$ {∵cor2π+θ=cosθ}

$= \cos^{-1}\left(\cos\frac{\pi}{6}\right)$

$= \frac{\pi}{6} \in [0 \cdot x]$

**Q2.** $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

**A.2.** $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \tan^{1}\left(\tan\frac{6\pi+\pi}{6}\right)$

$= \tan^{-1}\left(\tan\frac{6\pi}{6}+\frac{\pi}{6}\right)$

$$= \tan^{-1}\left(\tan \pi + \frac{\pi}{6}\right)$$

$$= \tan^{-1}\left(\tan \frac{\pi}{6}\right)$$ {∵ tan (π + θ ) = tan θ as tan b (+) we in 3rd quadrant)}

$$= \frac{\pi}{6} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$

**Prove that**

**Q3.** $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

**A.3.** Let $\sin^{-1}\frac{3}{5} = x$.Thun, $\sin x = \frac{3}{5}$: $\left( = \sqrt{1-\left(\frac{3}{5}\right)^2} = \right)$

Now , $\cos x = \sqrt{1-\sin^2 x} = \left|\sqrt{1-\frac{1}{25}}\right.$ $\{\because c\,os^2 x + \sin^2 x = 1\}$

$$= \sqrt{\frac{25-9}{25}}$$

$$= \sqrt{\frac{16}{26}}$$

$$= \frac{4}{5}.$$

So tan $x$ = $\frac{\sin x}{\cos x} = \frac{3/5}{4/5} = \frac{3}{4}$

Using.∴ $\tan 2x = \frac{2\tan 2}{1-\tan^2 x} = \frac{2\times 3/4}{1-(3/4)^2} = \frac{3/2}{1-9/16}$

$$= \frac{3/2}{16-9/16} = \frac{3}{2}\times\frac{16}{7} = \frac{24}{7}.$$

$$\Rightarrow \quad 2x = \tan^{-1}\frac{24}{7}.$$

$$\Rightarrow 2x\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{1}.$$

hence proved

**Q4.** $2\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

**A.4.** Let $\sin^{-1}\frac{8}{17} = x$ and $\sin^{-1}\frac{3}{5} = y$.

(N. then, $\sin x = \frac{8}{17}$ and $\sin y = \frac{3}{5}$.

Now, $\cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\left(\frac{8}{16}\right)^2} = \sqrt{1-\frac{6.4}{281}} = \sqrt{\frac{289-64}{286}}$

$= \sqrt{\frac{225}{289}} = \frac{15}{17}$

and $y = \sqrt{1-\sin^2 y} = \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{25-9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$.

So, $\tan x = \frac{\sin x}{\cos x}$ and $\tan y = \frac{\sin y}{\cos y}$

$= \frac{8/17}{15/17} = \frac{3/4}{4/5}$

$= \frac{8}{15} = \frac{3}{4}$

Using. $\tan(x+y) = \frac{\tan x + \tan y}{1-\tan x \tan y}$

$$\tan\left(\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5}\right) = \frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}}$$

$$\tan\ (\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \frac{\frac{8\times4+3\times15}{15\times4}}{\frac{15\times4-8\times3}{15\times4}} = \frac{32+45}{60-24} \Rightarrow \sin$$

$$\Rightarrow \sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$$

Hence proved.

**Q5.** $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

**A.5.** $cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65^\circ}$.

Let $\cos^{-1}\frac{4}{5}$ and $\cos^{-1}\frac{12}{13} = y$.

Then, $\cos x = \frac{4}{5}$ and $\cos y = \frac{12}{13}$.

Now, $\sin x = \sqrt{1-\cos^2 x} = \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{25-6}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$.

and $\sin y = \sqrt{1-\omega^2 y} = \sqrt{1-\left(\frac{12}{13}\right)^2} = \sqrt{\frac{1-144}{164}} = \sqrt{\frac{169-144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$

Using $\cos(x+y) = \cos x \cos y - \sin x \sin y$.

$$\Rightarrow \cos\left[\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}\right]=\frac{4}{5}\frac{12}{13}-\frac{3}{5}\times\frac{5}{13}$$

$$\Rightarrow \cos\left[\cos^{-1}\frac{4}{5}+\cot^{-1}\frac{12}{13}\right]=\frac{48-15}{65}=\frac{33}{65}$$

$$\Rightarrow \cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}.\phi$$

**Q6.** $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$

**A.6.** $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$

Let $\cos^{-1}\frac{12}{13}=x$ and $\sin^{-1}\frac{3}{5}=y.$

Thin, $\cos x=\frac{12}{13}$ and $\sin y=\frac{3}{5}$

Now, $\sin x=\sqrt{1-\cos^2 x}=\sqrt{1-\left(\frac{12^2}{13}\right)}=\sqrt{1-\frac{149}{169}}=\sqrt{\frac{169-144}{169}}=\sqrt{\frac{25}{169}}=\frac{5}{13}$

Cos$y$= $\sqrt{1-\sin^2 y}=\sqrt{1-\left(\frac{3}{5}\right)^2}=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{25-9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}.$

Using $\sin(x+y)=\sin x\cos y+\cos x\sin y.$

$$\Rightarrow sin\left[\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}\right]=\frac{5}{13}\times\frac{4}{5}+\frac{12}{13}\times\frac{3}{5}=\frac{20+36}{65}=\frac{56}{65}.$$

$$\Rightarrow \cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}.$$

**Q7.** $\tan^{-1}\frac{63}{16}=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$

**A.7.** Let $\sin^{-1}\frac{5}{13}=x$ and $\cos^{-1}\frac{3}{5}=y.$

Then, $\sin x=\frac{5}{13}$ and $\cos=\frac{3}{5}$

Now, $\cos x=\sqrt{1-\sin^2 x}=\sqrt{1-\left(\frac{5}{13}\right)^2}=\sqrt{\frac{1-25}{169}}=\sqrt{\frac{169-25}{161}}=\sqrt{\frac{144}{169}}=\frac{12}{13}$

and $\sin y=\sqrt{1-\cos^2 y}=\sqrt{1-\left(\frac{3}{5}\right)^2}=\sqrt{1-\frac{1}{25}}=\sqrt{\frac{25-1}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}.$

So, $\tan x=\frac{\sin x}{\cos x}$ *and* $\tan y=\frac{\sin y}{\cos y}$

$$=\frac{5/3}{12/1}\quad=\frac{4/5}{3/5}.$$

$$=\frac{5}{12} \quad =\frac{4}{3}.$$

Using $\tan(x+y)=\dfrac{\text{lan}\,x+\text{lan}\,y}{1-\tan x\tan y.}$

$$\tan\left(\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}\right)=\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\times\frac{4}{3}}=\frac{\frac{5\times3+4\times12}{12\times3}}{\frac{12\times3-5\times4}{12\times3}}$$

$$=\frac{15+48}{36-20}=\frac{63}{16}$$

$$\Rightarrow \sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{63}{16}.$$

Hence proved.

**Q8.** $\tan^{-1}\dfrac{1}{5}+\tan^{-1}\dfrac{1}{7}+\tan^{-1}\dfrac{1}{3}+\tan^{-1}\dfrac{1}{8}=\dfrac{\pi}{4}$

**A.8.** LH.S $=\left(\tan^{-1}\dfrac{1}{5}+\tan^{-1}\dfrac{1}{7}\right)+\left(\tan^{-1}\dfrac{1}{3}+\tan^{-1}\dfrac{1}{8}\right)$

$$=\tan^{-1}\left[\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}.\frac{1}{7}}\right]+\tan^{-1}\left[\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3},\frac{1}{8}}\right]\left\{\begin{array}{l}\because using\ \tan^{-1}x+\tan^{-1}y\\ \dfrac{x+y}{1-xy},xy<1\end{array}\right\}$$

$$=\tan^{-1}\left[\frac{\frac{7+5}{7\times5}}{\frac{7\times5-1}{7\times5}}\right]+\tan^{-1}\left[\frac{\frac{8+3}{5\times3}}{\frac{8\times3-1}{8\times3}}\right].$$

$$=\tan^{-1}\left(\frac{12}{35-1}\right)+\tan^{-1}\left(\frac{11}{24-1}\right)=\tan^{-1}\frac{12}{34}+\tan^{-1}\frac{11}{23}$$

$$=\tan^{-1}\frac{6}{17}+\tan^{-1}\frac{11}{23}$$

$$=\tan^{-1}\left(\frac{\frac{6}{17}+\frac{1}{23}}{1-\frac{6}{17}\times\frac{3}{23}}\right)=\tan^{-1}\left(\frac{\frac{6\times23+11\times11}{17\times23}}{\frac{7\times23-6\times11}{17\times23}}\right)$$

$$=\tan^{-1}\frac{138+187}{391-66}=\tan^{-1}\frac{325}{325}=\tan^{-1}1$$

$$=\tan^{-1}\left(\tan\frac{1}{4}\right)$$

$$=\frac{x}{4}=\ \text{R.H.S.}$$

**Prove that**

**Q9.** $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\frac{1-x}{1+x}, x \in [0,1]$

**A.9.** Let $\tan^{-1}\sqrt{x} = \theta, x \in [0,1]$

Then $\tan\theta = \sqrt{x}$

Squarig both sider , $x = \tan^2\theta$

So, R.H.S. $= \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\cos^{-1}\left(\frac{1-\text{lan}^2\theta}{1+\text{lan}^2\theta}\right)$

$\frac{1}{2}\cos^{-1}(\cos 20)\{\because qr2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{1}{2} \times 2\theta$

$= \theta$

$= \tan^{-1}\sqrt{x}$

= L.H.S.

**Q10.** $\cot^{1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$

**A.10.** $\cos^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{1}{4}\right)$

Hand

(h) we know that .

$1 = \cos^2 x + \sin^2 x$

So, 1 = cos2 $x/2$ + $\sin^2 x/2$ _____ (1)

And $\sin^2 x$= 2 sin $x$ cos $x$

So, sin x = sin $x/2$ cos $x/2$ _____ (2)

Adding eq$^n$ (1) and (2) we get,

$1+\sin x = \cos^2\frac{x}{2} + \sin^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2} = \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2$

$\{\because a^2 + b^2 - 2ab = (a-b)^2\}$.

Subtracting eq$^n$ (2) from (1) we get,

$1-\sin x = \cos^2\frac{x}{2} + \sin^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2} = \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2$ $\{\because a^2 + b^2 - 2ab = (a-b)^2\}$

So,

$$\text{L.H.S} = \cot^{-1}\left\{\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{(\cos x/2-\sin x/2)^2}}{\sqrt{(\cos x/2+\sin x/2)^2}-\sqrt{(\cos x/2-\sin x/2)^2}}\right\}$$

$$=\cot^{-1}\left\{\frac{\cos{}^{x}\!/_{2}+\sin{}^{x}\!/_{2}+\cos{}^{x}\!/_{2}-\sin{}^{x}\!/_{2}}{\cos{}^{x}\!/_{2}+\sin{}^{x}\!/_{2}-\cos{}^{x}\!/_{2}+\sin{}^{x}\!/_{2}}\right\}$$

$$=\cot^{-1}\left\{\frac{2\cos x/2}{2\sin x/2}\right\}-\cot^{-1}\left(\cot\frac{x}{2}\right)-\frac{x}{2}.$$

**Q11.** $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,-\frac{1}{\sqrt{2}}\le x\le 1\left[\text{Hint}:\text{Put } x=\cos 2\theta\right]$

**A.11.** Let $x=\cos 2\theta.\text{Then }\theta=\frac{1}{2}an^{-1}\frac{x}{x}$

$$\text{L.H.S. }\tan^{-1}=\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$$

$$=\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$

$$\left\{\begin{array}{l}\because 4\,\text{cor}\,2\theta=2\,\text{cor}^2\theta-1\\ \Rightarrow 1+\text{cor}^2\theta=2\cos^2\theta\\ \text{and }\cos 2\theta=1-2\sin^2\theta\\ \rightarrow 1-\text{cor}\,2\theta=2\sin^2\theta\end{array}\right.$$

$$=\tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$$

$$=\tan^{-1}\left(\frac{\frac{\sqrt{2}\cos\theta}{\sqrt{2}\cos\theta}-\frac{\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta}}{\frac{\sqrt{2}\cos\theta}{\sqrt{2}\cos\theta}+\frac{\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta}}\right)$$

$$=\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$=\tan^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right)\left\{\because \tan{}^{\pi}\!/_{4}=1\right\}$$

$$=\tan^{-1}\left\{\tan\left(\frac{x}{4}-\theta\right)\right\}\quad\left(\because \tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}\right)$$

$$=\tan\frac{\pi}{4}-\theta$$

$$=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x.$$

$= \text{R.H.S.}$

**Q12.** $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}-\sin^{-1}\frac{2\sqrt{2}}{3}$

**A.12.** L.H.S. $=\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}.\ \sin^{-1}\frac{1}{3}.$

(H)

$=\frac{9}{4}\left\{\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right\}$

$=\frac{9}{4}\left\{\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right\}\left\{\because \frac{x}{2}=\cos^{-1}x+\sin^{-1}x \quad \text{Put } x=\frac{1}{3}\right.$

$=\frac{9}{4}\left\{\cos^{-1}\frac{1}{3}\right\}=\frac{9}{4}\cdot x\left[\frac{\pi}{2}=\cos^{-1}\frac{1}{3}+\sin^{-1}\frac{1}{3} \rightarrow \frac{1}{2}-\sin^{-1}\frac{1}{3}=\cos^{-1}\frac{1}{3}\right.$

Lef $x=\cos^{-1}\frac{1}{3}$

So, $\sin x=\sqrt{1-\cos^2 x}=\sqrt{1-\left(\frac{1}{3}\right)^2}=\sqrt{1-\frac{1}{9}}=\sqrt{\frac{9-1}{9}}=\sqrt{\frac{8}{9}}=\frac{2\sqrt{2}}{3}$

So, L.H.S $=\frac{9}{4}\times\sin^{-1}\frac{2\sqrt{2}}{3}=$ R.H.S.

**Solve the following equations:**

**Q13.** $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$

**A.13.**. Given,

$(M)\ 2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow \tan^{-1}\frac{2\cos x}{1-\cos^2 x}=\tan^{-1}\frac{2}{\sin x}\left\{\begin{array}{l}\textit{using.}\\ \tan^{-1}2x=\frac{2x}{1-x^2}\end{array}\right\}$

$\Rightarrow \frac{2\cos x}{\sin^2 x}=\frac{2}{\sin x}\{\because\ 1-\cos^2 x=\sin^2 x \quad \Rightarrow 1=\sin^2 x+\cos^2 x\}$

$\Rightarrow \frac{\cos x}{\sin x}=1$

$\Rightarrow \quad \cot x=\cot\frac{x}{4}$

$\Rightarrow x=\frac{\pi}{4}.$

**Q14.** $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x,(x>0)$

**A.14.** Given,

(M)

$$\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$$

Lit $x=\tan\theta$. Then $\theta=\tan^{-1}x$. Bo we have,

$$\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)=\frac{1}{2}\tan^{-1}(\tan\theta)$$

$$\Rightarrow \tan^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{x}{4}\tan\theta}\right)=\frac{1}{2}\theta \quad \left\{\because \tan\frac{\pi}{4}=1\right\}.$$

$$\Rightarrow \tan^{-1}\left\{\tan\left(\frac{x}{4}-\theta\right)\right\}=\frac{\theta}{2}\left\{\because \frac{\tan x-\tan y}{1+\tan x\tan}=\tan(x-y)\right.$$

$$\Rightarrow \frac{\pi}{4}-\theta=\frac{\theta}{2}.$$

$$\Rightarrow \frac{\theta}{2}+\theta=\frac{x}{4}$$

$$\Rightarrow \frac{3\theta}{2}=\frac{\pi}{4}$$

$$\Rightarrow \theta=\frac{\pi}{4}\times\frac{2}{3}=\frac{\pi}{6}$$

So, $x=\tan\theta=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}.$

**Q15.** $\sin\left(\tan^{-1}x\right),|x|<1$ is equal to

(A) $\frac{x}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{\sqrt{1+x^2}}$ (D) $\frac{x}{\sqrt{1-x^2}}$

**A.15.** $\sin\left(\tan^{-1}x\right),|x|\nless.$

(M) .let $\theta=\tan^{-1}x$ . then $\tan\theta=x$ .

And $\sin(\tan^{-1}x).\sin\theta=\frac{1}{\cos\theta}=\frac{1}{\sqrt{1+\cot^2\theta}}$ $\{\because\cos^2\theta=1+\cot^2\theta\}$

$$=\frac{1}{\sqrt{1+\frac{1}{\tan^2}\theta}} \quad \left\{\because \cot\theta=\frac{1}{\tan\theta}.\right.$$

$$=\frac{1}{\sqrt{1+\frac{1}{x^2}}}=\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}=\frac{x}{\sqrt{1+x^2}}$$

$\because$Option D is correct.

**Q16.** $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$, then $x$ is equal to

(A) $0,\frac{1}{2}$ (B) $1,\frac{1}{2}$ (C) $0$ (D) $\frac{1}{2}$

**A.16.** $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}\rightarrow(1)$

(M) Let $x=\sin\theta. Then, \theta=\sin^{-1}x.$

Putting this in $q^n(1)$ we get

$$\sin^{-1}(1-x)-2\cdot\theta=\frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}(1-x)=\frac{\pi}{2}+2\theta$$

$$\Rightarrow \quad 1-x=\sin\left(\frac{\pi}{2}+2\theta\right)\left\{\sin\left(\frac{\pi}{2}+x\right)=\cos x\right\}$$

$$\Rightarrow \quad 1-x=\cos 2\theta$$

$$\Rightarrow \quad 1-x=1-2\sin^2\theta\cdot\left\{\cos 2x=1-2\sin^2 x\right\}$$

$$\Rightarrow 1-x=1-2x^2\cdot\{\sin\theta=x\}$$

$$\Rightarrow \quad 2x^2-x=0$$

$$\Rightarrow x(2x-1)=0$$

$$\Rightarrow so, \quad x=0 \quad x \quad 2x-1=0 \quad x \quad 2x=1\rightarrow x=\frac{1}{2}.$$

Putting $x=0$ in $q^n$ (1) .

L.H.S $=\sin^{-1}(1-0)-2\sin^{-1}0=\sin^{-1}\sin\frac{x}{2}-0=\frac{\pi}{2}=R.H.S.$

Putting $x=\frac{1}{2}$ in $q$ (1)

L.H.S $=\sin^{-1}\left(1-\frac{1}{2}\right)-2\sin^{-1}\frac{1}{2}=\sin^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\frac{1}{2}$

$$=\sin^{-1}\frac{1}{2}-2\sin^{-1}\frac{1}{2}$$

$$=-\sin^{-1}\frac{1}{2}=-\sin^{-1}\left(\sin\frac{\pi}{6}\right)=-\frac{\pi}{6}\neq \text{R.H.S.}$$

So, $x=0.$

Option (c) is correct.

**Q17.** $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\frac{x-y}{x=y}$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{3\pi}{4}$

**A.17.** $\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{x-y}{x+y} \quad \left\{-\tan^{-1}x-\tan^{-1}y=\frac{x-y}{1+xy}\right\}$

$$= \tan^{-1}\left\{\frac{\left(\frac{x}{y}\right)-\left(\frac{x-y}{x+y}\right)}{1+\frac{x}{y}\cdot\left(\frac{x-y}{x+y}\right)}\right\}$$

$$= \tan^{-1}\left\{\frac{\frac{x(x+y)-(x-y)\cdot y}{y(x+y)}}{\frac{y(x+y)+x(x-y)}{y(x+y)}}\right\}$$

$$= \tan^{-1}\left(\frac{x^2+xy-xy+y^2}{xy+y^2-x^2-xy}\right)$$

$$= \tan^{-1}\frac{x^2+y^2}{x^2+y^2}$$

$$= \tan^{-1}1$$

$$= \tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$= \frac{\pi}{4}.$$

$\because$ Option C is correct.