Question 1:

Find the principal value of $\sin^{-1}\left(-\dfrac{1}{2}\right)$

Answer

Let $\sin^{-1}\left(-\dfrac{1}{2}\right) = y.$ Then $\sin y = -\dfrac{1}{2} = -\sin\left(\dfrac{\pi}{6}\right) = \sin\left(-\dfrac{\pi}{6}\right).$

We know that the range of the principal value branch of $\sin^{-1}$ is

$\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ and $\sin\left(-\dfrac{\pi}{6}\right) = -\dfrac{1}{2}.$

Therefore, the principal value of $\sin^{-1}\left(-\dfrac{1}{2}\right)$ is $-\dfrac{\pi}{6}.$

Question 2:

Find the principal value of $\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

Answer

Let $\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = y.$ Then, $\cos y = \dfrac{\sqrt{3}}{2} = \cos\left(\dfrac{\pi}{6}\right).$

We know that the range of the principal value branch of $\cos^{-1}$ is

$[0, \pi]$ and $\cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$.

Therefore, the principal value of $\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ is $\dfrac{\pi}{6}$.

Question 3:

Find the principal value of $\operatorname{cosec}^{-1}(2)$

Answer

Let cosec^{-1} (2) = y. Then,

$$\operatorname{cosec} y = 2 = \operatorname{cosec}\left(\frac{\pi}{6}\right).$$

We know that the range of the principal value branch of cosec^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}.$

Therefore, the principal value of cosec^{-1} (2) is $\frac{\pi}{6}$.

Question 4:

Find the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$

Answer

Let $\tan^{-1}\left(-\sqrt{3}\right) = y$. Then, $\tan y = -\sqrt{3} = -\tan\frac{\pi}{3} = \tan\left(-\frac{\pi}{3}\right).$

We know that the range of the principal value branch of tan^{-1} is

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{3}\right)$ is $-\sqrt{3}.$

Therefore, the principal value of $\tan^{-1}\left(\sqrt{3}\right)$ is $-\frac{\pi}{3}.$

Question 5:

Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$

Answer

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$. Then, $\cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right).$

We know that the range of the principal value branch of cos^{-1} is

$[0, \pi]$ and $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}.$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}.$

Question 6:

Find the principal value of $\tan^{-1}(-1)$

Answer

$$\tan y = -1 = -\tan\left(\frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right).$$

Let $\tan^{-1}(-1) = y$. Then,

We know that the range of the principal value branch of $\tan^{-1}$ is

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{4}\right) = -1.$

$$\tan^{-1}(-1) \text{ is } -\frac{\pi}{4}.$$

Therefore, the principal value of

Question 7:

$$\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

Find the principal value of

Answer

Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$. Then, $\sec y = \frac{2}{\sqrt{3}} = \sec\left(\frac{\pi}{6}\right).$

We know that the range of the principal value branch of $\sec^{-1}$ is

$\left[0, \pi\right] - \left\{\frac{\pi}{2}\right\}$ and $\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}.$

$$\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) \text{ is } \frac{\pi}{6}.$$

Therefore, the principal value of

Question 8:

$$\cot^{-1}\left(\sqrt{3}\right)$$

Find the principal value of

Answer

Let $\cot^{-1}\left(\sqrt{3}\right) = y$. Then, $\cot y = \sqrt{3} = \cot\left(\frac{\pi}{6}\right).$

We know that the range of the principal value branch of $\cot^{-1}$ is $(0, \pi)$ and

$$\cot\left(\frac{\pi}{6}\right) = \sqrt{3}.$$

Therefore, the principal value of $\cot^{-1}\left(\sqrt{3}\right)$ is $\frac{\pi}{6}$.

Question 9:

Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Answer

Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y$. Then, $\cos y = -\frac{1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)$.

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$ and

$$\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}.$$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

Question 10:

Find the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

Answer

Let $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right) = y$. Then, $\operatorname{cosec} y = -\sqrt{2} = -\operatorname{cosec}\left(\frac{\pi}{4}\right) = \operatorname{cosec}\left(-\frac{\pi}{4}\right)$.

We know that the range of the principal value branch of $\operatorname{cosec}^{-1}$ is

$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\} \text{ and } \operatorname{cosec}\left(-\frac{\pi}{4}\right) = -\sqrt{2}.$$

Therefore, the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$ is $-\frac{\pi}{4}$.

Question 11:

Find the value of
$$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

Answer

Let $\tan^{-1}(1) = x$. Then, $\tan x = 1 = \tan\frac{\pi}{4}$.

$\therefore \tan^{-1}(1) = \frac{\pi}{4}$

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$. Then, $\cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$.

$\therefore \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

Let $\sin^{-1}\left(-\frac{1}{2}\right) = z$. Then, $\sin z = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right)$.

$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\therefore \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$

$= \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$

Question 12:

Find the value of
$$\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$$

Answer

Let $\cos^{-1}\left(\dfrac{1}{2}\right) = x$. Then, $\cos x = \dfrac{1}{2} = \cos\left(\dfrac{\pi}{3}\right)$.

$\therefore \cos^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{3}$

Let $\sin^{-1}\left(\dfrac{1}{2}\right) = y$. Then, $\sin y = \dfrac{1}{2} = \sin\left(\dfrac{\pi}{6}\right)$.

$\therefore \sin^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{6}$

$\therefore \cos^{-1}\left(\dfrac{1}{2}\right) + 2\sin^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{3} + \dfrac{2\pi}{6} = \dfrac{\pi}{3} + \dfrac{\pi}{3} = \dfrac{2\pi}{3}$

Question 13:

Find the value of if $\sin^{-1} x = y$, then

(A) $0 \le y \le \pi$ (B) $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$

(C) $0 < y < \pi$ (D) $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$

Answer

It is given that $\sin^{-1} x = y$.

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$.

Therefore, $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$.

Question 14:

Find the value of $\tan^{-1} \sqrt{3} - \sec^{-1}(-2)$ is equal to

(A) π (B) $-\dfrac{\pi}{3}$ (C) $\dfrac{\pi}{3}$ (D) $\dfrac{2\pi}{3}$

Answer

Let $\tan^{-1}\sqrt{3} = x$. Then, $\tan x = \sqrt{3} = \tan\dfrac{\pi}{3}$.

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$.

$\therefore \tan^{-1}\sqrt{3} = \dfrac{\pi}{3}$

Let $\sec^{-1}(-2) = y$. Then, $\sec y = -2 = -\sec\left(\dfrac{\pi}{3}\right) = \sec\left(\pi - \dfrac{\pi}{3}\right) = \sec\dfrac{2\pi}{3}$.

We know that the range of the principal value branch of $\sec^{-1}$ is $\left[0,\pi\right] - \left\{\dfrac{\pi}{2}\right\}$.

$\therefore \sec^{-1}(-2) = \dfrac{2\pi}{3}$

Hence, $\tan^{-1}\left(\sqrt{3}\right) - \sec^{-1}(-2) = \dfrac{\pi}{3} - \dfrac{2\pi}{3} = -\dfrac{\pi}{3}$