Miscellaneous exercise

Q1. Write down a unit vector in XY-plane making an angle of 30° with the positive direction of x-axis.

A.1. Let $\vec{r}$ be unit vector in the XY-plane then, $\vec{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

θ is the angle made by the unit vector with the positive direction of the X-axis.

Then, $\theta = 30^0$

$$\vec{r} = \cos 30^0\,\hat{i} + \sin 30^0\,\hat{j}$$

$$= \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$$

$\therefore$ ReQ.uired unit vector $= \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

Q2. Find the scalar components and magnitude of the vector joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$

A.2. Given,

Point $P(x_1, y_1, z_1)$ & $Q(x_2, y_2, z_2)$

$\overrightarrow{PQ}$ = Position vector of Q.- Position vector of P

$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

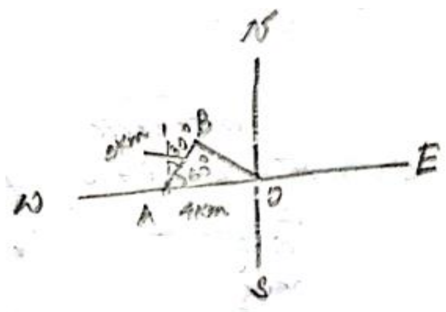
Therefore, $\left|\overrightarrow{PQ}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Thus, the scalar component is $(x_2 - x_1)(y_2 - y_1)(z_2 - z_1)$

& magnitude $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Q3. A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

A.3.



Let, 0 be initial and B be the final position of girl.

Then, $\overrightarrow{OA} = -4\hat{i}$

$$\overrightarrow{AB} = \hat{i}\left|\overrightarrow{AB}\right|\cos 60^\circ + \hat{j}\left|\overrightarrow{AB}\right|\sin 60^\circ$$

$= \hat{i}\, 3 \times \frac{1}{2} + \hat{j}\, 3 \times \frac{\sqrt{3}}{2}$

$= \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

By the triangle law of vector addition, we have

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= \left(-4\hat{i}\right) + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right)$

$= \left(-4 + \frac{3}{2}\right)\hat{i} + \left(\frac{3\sqrt{3}}{2}\right)\hat{j}$

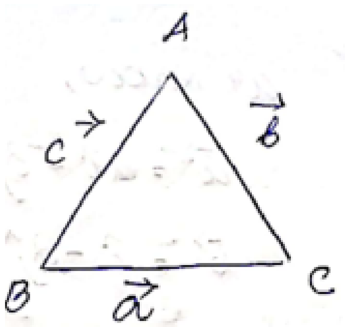
$= \left(\frac{-8+3}{2}\right)\hat{i} + \left(\frac{3\sqrt{3}}{2}\right)\hat{j}$

$= \frac{-5}{3}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

$\therefore$ The girl's displacement from her initial point of departure is $= \frac{-5}{3}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

Q4. If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$ Justify your answer.

A.4. Let us take a $\triangle ABC$,which $\overrightarrow{CB} = \vec{a}, \overrightarrow{CA} = \vec{b}$ & $\overrightarrow{AB} = \vec{c}$



So, by triangle law of vector addition, we have $\vec{a} = \vec{b} + \vec{c}$

And, we know that $|\vec{a}| |\vec{b}|$ & $|\vec{c}|$ represent, the sides of $\triangle ABC$

Also, it is known that the sum of the length of any slides of a triangle is greater than the third side. $|\vec{a}| < |\vec{b}| + |\vec{c}|$

Hence, it is not true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$

Q5. Find the value of x for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

A.5. Given,

$x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

So, $\left|x(\hat{i} + \hat{j} + \hat{k})\right| = 1$

Now, $\left|x(\hat{i} + \hat{j} + \hat{k})\right| = 1$

$\Rightarrow \sqrt{x^2 + x^2 + x^2} = 1$

$\Rightarrow \sqrt{3x^2} = 1$

$\Rightarrow \sqrt{3}x = 1$

$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$

$\therefore$ ReQ.uired value of x is $\pm\frac{1}{\sqrt{3}}$

Q6. Find a vector of magnitude 5 units and parallel to the resultant of the vectors $a = 2\hat{i} + 3\hat{j} - \hat{k}$ and $b = \hat{i} - 2\hat{j} + \hat{k}$

A.6. We know,

$$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$$

$$\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$$

Let, $\vec{c}$ be the resultant of $\vec{a}$ and $\vec{b}$

Then,

$$\vec{c} = \vec{a} + \vec{b}$$

$$= \left(2\hat{i} + 3\hat{j} - \hat{k}\right) + \left(\hat{i} - 2\hat{j} + \hat{k}\right)$$

$$= (2+1)\hat{i} + (3-2)\hat{j} + (-1+1)\hat{k}$$

$$= 3\hat{i} + \hat{j}$$

$$|\vec{c}| = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

$$\therefore \hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{3\hat{i} + \hat{j}}{\sqrt{10}}$$

Therefore, the vector of magnitude 5 units and parallel to the resultants of vectors $\vec{a}$ and $\vec{b}$ is

$$\pm 5.\hat{c}$$

$$= \pm 5.\frac{3\hat{i} + \hat{j}}{\sqrt{10}}$$

$$= \pm 5.\frac{3\hat{i}}{\sqrt{10}} \pm \frac{5\hat{j}}{\sqrt{10}}$$

$$= \pm\frac{5.3\sqrt{10}}{\sqrt{10}.\sqrt{10}}\hat{i} \pm \frac{5\sqrt{10}}{\sqrt{10}.\sqrt{10}}\hat{j}$$

$$= \pm\frac{5.3\sqrt{10}}{10}\hat{i} \pm \frac{5\sqrt{10}}{10}\hat{j}$$

$$= \pm\frac{3\sqrt{10}}{2}\hat{i} \pm \frac{\sqrt{10}}{2}\hat{j}$$

Q7. If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = 2\vec{i} - \vec{j} + 3\vec{k}$ and $c = \vec{i} - 2\vec{j} + \vec{k}$ **find a unit vector parallel to the vector** $2\vec{a} - \vec{b} + 3\vec{c}$

A.7. Given,

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$$

Then,

$$2\vec{a} - \vec{b} + 3\vec{c} = 2\left(\hat{i} + \hat{j} + \hat{k}\right) - \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + 3\left(\hat{i} - 2\hat{j} + \hat{k}\right)$$

$$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 2\hat{i} + \hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$$

$$= -3\hat{j} + 3\hat{i} + 2\hat{k}$$

$$= 3\hat{i} - 3\hat{j} + 2\hat{k}$$

$$\left|2\vec{a} - \vec{b} + 3\vec{c}\right| = \sqrt{3^2 + (-3)^2 + (2)^2}$$

$$= \sqrt{9 + 9 + 4} = \sqrt{22}$$

Hence, the unit vector is,

$$\frac{2\vec{a} - \vec{b} + 3\vec{c}}{\left|2\vec{a} - \vec{b} + 3\vec{c}\right|} = \frac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{22}} = \frac{3\hat{i}}{\sqrt{22}} - \frac{3\hat{j}}{\sqrt{22}} + \frac{2\hat{k}}{\sqrt{22}}$$

Q8. Show that the points A (1, –2, –8), B (5, 0, –2) and C (11, 3, 7) are collinear and find the ratio in which B divides AC.

A.8. Given,

$$A(1, -2, -8)$$

$$B(5, 0, -2)$$

$$C(11, 3, 7)$$

Now,

$\overrightarrow{AB} = (5-1)\hat{i} + (0-(-2))\hat{j} + (-2-(-8))\hat{k}$

$= 4\hat{i} + 2\hat{j} + 6\hat{k}$

$\overrightarrow{BC} = (11-5)\hat{i} + (3-0)\hat{j} + (7-(-2))\hat{k}$

$= 6\hat{i} + 3\hat{j} + 9\hat{k}$

$\overrightarrow{AC} = (11-1)\hat{i} + (3-(-2))\hat{j} + (7-(-8))\hat{k}$

$= 10\hat{i} + 5\hat{j} + 15\hat{k}$

$|\overrightarrow{AB}| = \sqrt{4^2 + 2^2 + 6^2} = \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{14}$

$|\overrightarrow{BC}| = \sqrt{6^2 + 3^2 + 9^2} = \sqrt{36+9+81} = \sqrt{126} = 3\sqrt{14}$

$|\overrightarrow{AC}| = \sqrt{10^2 + 5^2 + 15^2} = \sqrt{100+25+225} = \sqrt{350} = 5\sqrt{14}$

$\therefore |\overrightarrow{AC}| = 5\sqrt{14}$

$|\overrightarrow{AB}| + |\overrightarrow{BC}| = 2\sqrt{14} + 3\sqrt{14} = 5\sqrt{14}$

$|\overrightarrow{AC}| = |\overrightarrow{AB}| + |\overrightarrow{BC}|$

Thus, A,B and C are collinear.

Let, $\lambda : 1$ be the ratio that point B divides AC.

We have,

$$\overrightarrow{OB} = \frac{\lambda\overrightarrow{OC} + \overrightarrow{OA}}{\lambda + 1}$$

$$5\hat{i} - 2\hat{k} = \frac{\lambda(11\hat{i} + 3\hat{j} + 7\hat{k}) + (\hat{i} - 2\hat{j} - 8\hat{k})}{\lambda + 1}$$

$(5\hat{i} - 2\hat{k})(\lambda + 1) = 11\lambda\hat{i} + 3\lambda\hat{j} + 7\lambda\hat{k} + \hat{i} - 2\hat{j} - 8\hat{k}$

$5(\lambda+1)\hat{i} - 2(\lambda+1)\hat{k} = (11\lambda+1)\hat{i} + (3\lambda-2)\hat{j} + (7\lambda-8)\hat{k}$

On eQ.uating the corresponding component , we get

$5(\lambda+1) = 11\lambda + 1$

$5\lambda + 5 = 11\lambda + 1$

$5 - 1 = 11\lambda - 5\lambda$

$4 = 6\lambda$

$\therefore \lambda = \frac{4}{6} = \frac{2}{3}$

Hence, point B divides AC in the ratio $2:3$

Q9. Find the position vector of a point R which divides the line joining the two points P and Q whose position vectors are $\left(2\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-3\vec{b}\right)$ externally in the ratio 1 : 2. Also, show that P is the middle point of line segment RQ.

A9. Given,

$$P(2\vec{a}+\vec{b}) i.e, OP = 2\vec{a}+\vec{b}$$

$$Q(\vec{a}-3\vec{b}) i.e, OQ = \vec{a}-3\vec{b}$$

It is given that point R divides a line segment joining two points P and Q.

externally in the ratio 1:2 Then,

$$\overrightarrow{OR} = \frac{2(2\vec{a}+\vec{b})-(\vec{a}-3\vec{b})}{2-1}$$

$$= \frac{4\vec{a}+\overrightarrow{2b}-\vec{a}+3\vec{b}}{1}$$

$$\overrightarrow{OR} = 3\vec{a}+5\vec{b}$$

$\therefore$ Position vector of the mid-point of RQ.

$$= \frac{\overrightarrow{OQ}+\overrightarrow{OR}}{2}$$

$$= \frac{(\vec{a}-3\vec{b})+(3\vec{a}+5\vec{b})}{2}$$

$$= \frac{\vec{a}-3\vec{b}+3\vec{a}+5\vec{b}}{2}$$

$$= \frac{4\vec{a}-2\vec{b}}{2} = 2\vec{a}-\vec{b} = \overrightarrow{OR} \quad \textit{Hence proved}$$

Q10. Two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area.

A.10. Given,

Adjacent sides of parallelogram are

$$\vec{a} = 2\hat{i}-4\hat{j}+5\hat{k}$$

$$\vec{b} = \hat{i}-2\hat{j}-3\hat{k}$$

$\therefore$ Diagonal of parallelogram = $\vec{a}+\vec{b}$

$$\Rightarrow \vec{a}+\vec{b} = (2+1)\hat{i}+(-4+(-2))\hat{j}+(5+(-3))\hat{k}$$

$$= 3\hat{i}-6\hat{j}+2\hat{k}$$

Thus, the unit vector parallel to diagonal

$$\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|} = \frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{3^2+(-6)^2+(2)^2}} = \frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{9+36+4}}$$

$$= \frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{49}} = \frac{3\hat{i}-6\hat{j}+2\hat{k}}{7}$$

$$= \frac{3\hat{i}}{7} - \frac{6\hat{j}}{7} + \frac{2\hat{k}}{7}$$

$\because$ Area of parallelogram ABCD = $|\vec{a}+\vec{b}|$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix}$$

$$= \hat{i}(12+10) - \hat{j}(-6-5) + \hat{k}(-4+4)$$

$$= 22\hat{i} + 11\hat{j} + 0$$

$$= 22\hat{i} + 11\hat{j} = 11(2\hat{i} + 1\hat{j})$$

$$\therefore |\vec{a}+\vec{b}| = 11\sqrt{(2)^2+1^2}$$

$$= 11\sqrt{5}$$

$\because$ Therefore, area of parallelogram is $11\sqrt{5}$ sQ..unit.

Q11. Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$,

A.11. Let, a vector be equally inclined to axis OX,OY and OZ at angle α

Then, the direction cosine of the vector are $\cos\alpha, \cos\alpha \,\&\, \cos\alpha$

Now,

$$\cos^2\alpha, \cos^2\alpha \,\&\, \cos^2\alpha = 1$$

$$\Rightarrow 3\cos^2\alpha = 1$$

$$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$$

Therefore, the direction cosine of vector, vector are inclined equally to axis, are

$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$,

Q12. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. **Find a vector** $\vec{d}$ **which is perpendicular to both** $\vec{a}$ **and** $\vec{b}$ **and** $\vec{c}.\vec{d}$ **= 15**

A.12. Given,

$$\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$$

$$\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$$

$$\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$$

Let, $\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$

Since, $\vec{d}$ is perpendicular to both $\vec{a} \& \vec{b}$

$$\vec{d}.\vec{a} = 0$$

$$\Rightarrow d_1 + d_2 4 + d_3 2 = 0 ------(1)$$

$$\vec{d}.\vec{b} = 0$$

$$\Rightarrow d_1 3 + d_2(-2) + d_3(7) = 0$$

$$\Rightarrow d_1 3 - 2d_2 + 7d_3 = 0 ------(2)$$

We know,

$$\vec{c}.\vec{d} = 15$$

$$\Rightarrow 2d_1 - d_2 + 4d_3 = 15 ------(3)$$

$From,$ (1)

$$d_1 + 4d_2 + 2d_3 = 0$$

$$d_1 = -4d_2 - 2d_3$$

Putting this value in (3) we get

$$\Rightarrow 2(-4d_2 - 2d_3) - d_2 + 4d_3 = 15$$

$$\Rightarrow -8d_2 - 4d_3 - d_2 + 4d_3 = 15$$

$$\Rightarrow -9d_2 = 15$$

$$\Rightarrow d_2 = \frac{15}{9} = -\frac{5}{3}$$

Putting $d_1 \& d_2$ value in (2), we get

$$\Rightarrow 3d_1 - 2d_2 + 7d_3 = 0$$

$$\Rightarrow 3(-4d_2 - 2d_3) - 2\left(-\frac{5}{3}\right) + 7d_3 = 0$$

$$\Rightarrow -12 \times \left(-\frac{5}{3}\right) - 6d_3 + \frac{10}{3} + 7d_3 = 0$$

$$\Rightarrow 20 + d_3 + \frac{10}{3} = 0$$

$$\Rightarrow d_3 = -20 - \frac{10}{3} = \frac{-60-10}{3} = \frac{-70}{3}$$

$$Now, d_1 = -4 \times -\frac{5}{3} - 2 \times -\frac{70}{3}$$

$$= \frac{20}{3} + \frac{140}{3} = \frac{160}{3}$$

$$\therefore d_1 = \frac{160}{3}, d_2 = -\frac{5}{3}, d_3 = \frac{-70}{3}$$

$$\vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j}\frac{-70}{3}\hat{k} = \frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k})$$

$\therefore$ The reQ.uired vector is $\frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k})$

Q13. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

A.13. $(2\hat{i}+4\hat{j}-5\hat{k})+(\lambda\hat{i}+2\hat{j}+3\hat{k})=(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$

The unit vector along $(2\hat{i}+4\hat{j}-5\hat{k})+(\lambda\hat{i}+2\hat{j}+3\hat{k})$ is given as;

$$= \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}}$$

$$= \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{4+4\lambda+\lambda^2+36+4}}$$

$$= \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{44+4\lambda+\lambda^2}}$$

By Q.uestion , scalar product of $\left(\hat{i}+\hat{j}+\hat{k}\right)$ with this unit vector is 1.

$$\left(\hat{i}+\hat{j}+\hat{k}\right).\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{44+4\lambda+\lambda^2}}=1$$

$$\Rightarrow \frac{(2+\lambda)+6-2}{\sqrt{44+4\lambda+\lambda^2}}=1$$

$$\Rightarrow (2+\lambda)+4=\sqrt{44+4\lambda+\lambda^2}$$

$$\Rightarrow 2+\lambda+4=\sqrt{44+4\lambda+\lambda^2}$$

$$\Rightarrow (\lambda+6)^2=44+4\lambda+\lambda^2$$

$$\Rightarrow \lambda^2+12\lambda+36=44+4\lambda+\lambda^2$$

$$\Rightarrow \lambda^2+12\lambda+36=\lambda^2+4\lambda+44$$

$$\Rightarrow 12\lambda-4\lambda=44-36$$

$$\Rightarrow 8\lambda=8$$

$$\Rightarrow \lambda=\frac{8}{8}=1$$

$\therefore$ Hence, the value of λ is 1.

Q14. If are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ -is equally inclined to $a, b,$ and $\vec{c}$.

A.14. Given that $\vec{a}, \vec{b} \,\&\, \vec{c}$ are mutually perpendicular vectors, we have

$$\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$$

$$\left|\vec{a}\right|=\left|\vec{b}\right|=\left|\vec{c}\right|$$

Let, vector $\vec{a}+\vec{b}+\vec{c}$ be inclined to $\vec{a}, \vec{b} \,\&\, \vec{c}$ at angles, $\theta_1, \theta_2 \,\&\, \theta_3$ respectively.

$$\therefore \cos\theta_1=\frac{(\vec{a}+\vec{b}+\vec{c}).\vec{a}}{\left|\vec{a}+\vec{b}+\vec{c}\right|\left|\vec{a}\right|}=\frac{\vec{a}.\vec{a}+\vec{b}.\vec{a}+\vec{c}.\vec{a}}{\left|\vec{a}+\vec{b}+\vec{c}\right|\left|\vec{a}\right|}$$

$$=\frac{\left|\vec{a}\right|^2}{\left|\vec{a}+\vec{b}+\vec{c}\right|\left|\vec{a}\right|}\left[\vec{b}.\vec{a}=\vec{c}.\vec{a}=0\right]$$

$$=\frac{\left|\vec{a}\right|}{\left|\vec{a}+\vec{b}+\vec{c}\right|}$$

$$\therefore \cos\theta_2 = \frac{(a+b+c).b}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|}$$

$$= \frac{\vec{a}.\vec{b}+\vec{b}.\vec{b}+\vec{b}.\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|}\left[\vec{a}.\vec{b}=\vec{b}.\vec{c}=0\right]$$

$$= \frac{|\vec{b}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|} = \frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\therefore \cos\theta_3 = \frac{(\vec{a}+\vec{b}+\vec{c}).\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|}$$

$$= \frac{\vec{a}.\vec{c}+\vec{b}.\vec{c}+\vec{c}.\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|}\left[\vec{a}.\vec{c}=\vec{b}.\vec{c}=0\right]$$

$$= \frac{|\vec{c}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|} = \frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$now, as, |\vec{a}| = |\vec{b}| = |\vec{c}|,$$

$$\cos\theta_1 = \cos\theta_2 = \cos\theta_3$$

$$\therefore \theta_1 = \theta_2 = \theta_3$$

Therefore, the vector $\left(\vec{a}+\vec{b}+\vec{c}\right)$ are equally inclined to $\vec{a}, \vec{b} \,\&\, \vec{c}$.

Q.15. Prove that $\left(\vec{a}+\vec{b}\right).\left(\vec{a}+\vec{b}\right) = |\vec{a}|^2 + |\vec{b}|^2$ **if and only if** $\vec{a}, \vec{b}$ **are perpendicular given** $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}$**.**

A15. $(\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$

$$\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{b}+\vec{b}.\vec{b} = |\vec{a}|^2 + |\vec{b}|^2$$

(Distributive of scalar product over addition)

$$\Rightarrow |\vec{a}|^2 + 2\vec{a}.\vec{b} + |\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$$

(Scalar product is commutative , $\vec{a}.\vec{b} = \vec{b}.\vec{a}$)

$$\Rightarrow 2\vec{a}.\vec{b} = |\vec{a}|^2 - |\vec{a}|^2 + |\vec{b}|^2 - |\vec{b}|^2$$

$$\Rightarrow 2\vec{a}.\vec{b} = 0$$

$$\Rightarrow \vec{a}.\vec{b} = 0$$

$\therefore$ Therefore, $\vec{a} \,\&\, \vec{b}$ are perpendicular.

Q16. Choose the correct answer:

If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$ then $\vec{a}.\vec{b} \geq 0$ only when:

(A) $0 < \theta < \frac{\pi}{2}$

(B) $0 \leq \theta \leq \frac{\pi}{2}$

(C) $0 < \theta < \pi$

(D) $0 \leq \theta \leq \pi$

A.16. Let, θ in triangle between two vector $\vec{a} \& \vec{b}$

Then, without loss of generality, $\vec{a} \& \vec{b}$ are non-zero vector so that $|\vec{a}| \& |\vec{b}|$ are positive.

We know, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

So, $\vec{a}.\vec{b} \geq 0$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta \geq 0$

$\cos\theta \geq 0 \left[\therefore |\vec{a}| \& |\vec{b}| arepositive\right]$

$0 \leq \theta \leq \frac{\pi}{2}$

Therefore, $\vec{a}.\vec{b} \geq 0$, when $0 \leq \theta \leq \frac{\pi}{2}$

Hence, the correct answer is B.

Q17. Choose the correct answer:

Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if

(A) θ = $\frac{\pi}{4}$

(B) θ = $\frac{\pi}{3}$

(C) θ = $\frac{\pi}{2}$

(D) θ = $\frac{\pi}{3}$

A.17. Let, $a \& b$ be two unit vectors and θ be the angle between them.

Then, $|\vec{a}| = |\vec{b}| = 1$

Now, $\vec{a} + \vec{b}$ is a unit vector if $|\vec{a} + \vec{b}| = 1$

$|\vec{a} + \vec{b}| = 1$

$(\vec{a} + \vec{b})^2 = 1$

$(\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = 1$

$\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{b}.\vec{a} + \vec{b}.\vec{b} = 1$

$|\vec{a}|^2 + 2\vec{a}.\vec{b} + |\vec{b}|^2 = 1$

$1^2 + 2|\vec{a}|.|\vec{b}|\cos\theta + 1^2 = 1$

$1 + 2.1.1\cos\theta + 1 = 1$ [$\therefore$ $\vec{a} \& \vec{b}$ is unit vector.]

$2\cos\theta = 1 - 2$

$\cos\theta = \frac{-1}{2} = \frac{2\pi}{3}$

$\therefore \theta = \frac{2\pi}{3}$

Therefore, the correct answer is (D)

Q18. Choose the correct answer:

The value of $\hat{i}.\left(\hat{j} \times \hat{k}\right) + \hat{j}.\left(\hat{i} \times \hat{k}\right) + \hat{k}\left(\hat{i} \times \hat{j}\right)$ **is:**

(A) 0

(B) -1

(C) 1

(D) 3

A18. $\hat{i}\left(\hat{j} \times \hat{k}\right) + \hat{j}\left(\hat{i} \times \hat{k}\right) + \hat{k}\left(\hat{i} \times \hat{j}\right)$

$= \hat{i}.\hat{i} + \hat{j}\left(-\hat{j}\right) + \hat{k}.\hat{k}$

$= 1 - \hat{j}.\hat{j} + 1$

$= 1 - 1 + 1$

$= 1$

Therefore, the correct answer is (C)

Q19. If θ be the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$ when θ is equal to:

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) π

A19. Let, θ be angle between two vector $\vec{a} \& \vec{b}$.

Then, without loss of generality, $\vec{a} \& \vec{b}$ are non-zero vectors, so that $\vec{a} \& \vec{b}$ are positive.

$$|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$$

$$|\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta$$

$$\cos\theta = \sin\theta \left[|\vec{a}| \& |\vec{b}| are positive\right]$$

$$\tan\theta = 1 = \frac{\pi}{4} \therefore \theta = \frac{\pi}{4}.$$

$\therefore$ Therefore, the correct answer is B.